4.4

Solving Problems Using Obtuse Triangles

YOU WILL NEED
- calculator
- ruler

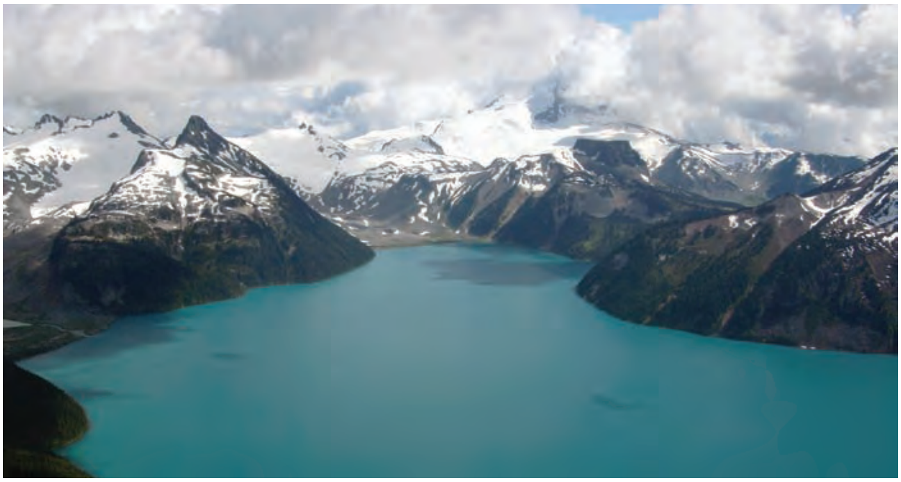
GOAL

Solve problems that can be modelled by one or more obtuse triangles.

EXPLORE...
- The cross-section of a canal has two slopes and is triangular in shape. The angles of inclination for the slopes measure 28° and 49°. When the canal is full of water, the length of one of the slopes is 12 m. What is the width of the surface of the water when the canal is full?

LEARN ABOUT the Math

A surveyor in a helicopter would like to know the width of Garibaldi Lake in British Columbia. When the helicopter is hovering at 1610 m above the forest, the surveyor observes that the angles of depression to two points on opposite shores of the lake measure 45° and 82°. The helicopter and the two points are in the same vertical plane.



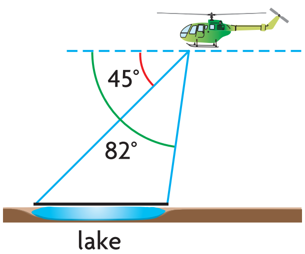
This view of the south part of Garibaldi Lake was captured from the Panorama Ridge trail.

? What is the width of Garibaldi Lake?

EXAMPLE 1 Visualizing a triangle to solve a problem

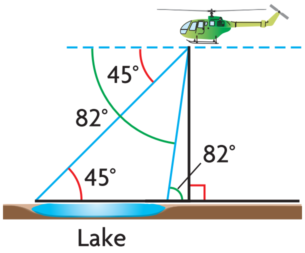
Determine the width of the lake, to the nearest metre.

Spencer's Solution: Creating right triangles



I drew a diagram of the helicopter over the forest, with its sight lines.

Angles of depression are always measured against the horizontal, so I drew a horizontal line and placed the angles.

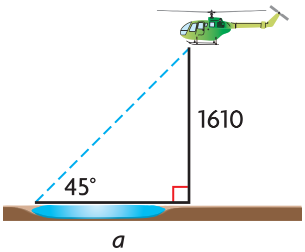


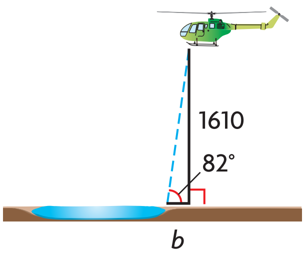
Because the lake is also horizontal, the alternate interior angles are equal.

I drew the altitude of the helicopter on the triangle. I realized that the sight lines form two right triangles.

Let a represent the distance from one end of the lake to the point directly below the helicopter.

Let b represent the distance from the other side of the lake to the point directly below the helicopter.





I redrew each right triangle.

$$\tan 45° = \frac{1610}{a}$$
$$a \tan 45° = a\left(\frac{1610}{a}\right)$$
$$a = \frac{1610}{\tan 45°}$$
$$a = 1610$$

$$\tan 82° = \frac{1610}{b}$$
$$b \tan 82° = b\left(\frac{1610}{b}\right)$$
$$b = \frac{1610}{\tan 82°}$$
$$b = 226.270\ldots$$

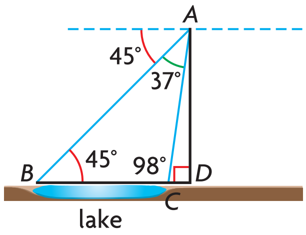
For both right triangles, the measure of an angle and the length of its opposite side are known. The unknown base is the adjacent side of the angle. I used the tangent ratio to determine the length of the base in each triangle.

Width of lake $= a - b$
Width of lake $= 1610 - 226.270\ldots$
Width of lake $= 1383.730\ldots$ m

Since a represents the width of the lake and a small piece of land beneath the helicopter, and b represents the small piece of land beneath the helicopter, the width of the lake is $a - b$.

The width of the lake is about 1384 m.

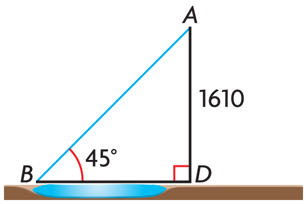
Emily's Solution: Using the sine law



I drew a diagram to represent the situation.

I used parallel lines to determine the measure of ∠ *B*. Then I calculated the remaining angle in the base to be 98°, since the measures of angles in a triangle add to 180°.

I calculated the angle at the helicopter, between the sight lines, by subtraction.



In $\triangle ABD$:

$$\sin 45° = \frac{1610}{AB}$$

$$AB \sin 45° = \left(\frac{1610}{AB}\right)AB$$

$$AB = \frac{1610}{\sin 45°}$$

$$AB = 2276.883\ldots$$

I used the primary trigonometric ratios to determine the length of *AB*. *AB* is a side in both $\triangle ABD$ and $\triangle ABC$.

In $\triangle ABC$:

$$\frac{AB}{\sin 98°} = \frac{BC}{\sin 37°}$$

$$\sin 37°\left(\frac{2276.883\ldots}{\sin 98°}\right) = BC$$

$$1383.729\ldots = BC$$

I used the sine law to determine the width of the lake, *BC*.

The width of the lake is about 1384 m.

Reflecting

A. Could Emily have used the cosine law to calculate the width of the lake?

B. Does Emily need to worry about the ambiguous case when using the sine law in this situation? Explain.

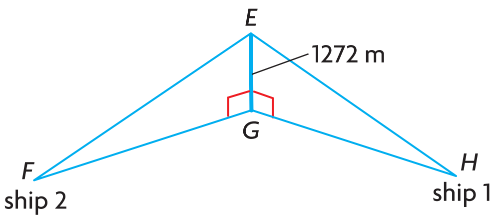
APPLY the Math

EXAMPLE 2 Solving a 3-D problem

A wind turbine called the Eye of the Wind is located at the top of Grouse Mountain in Vancouver. Rae is standing in the viewing pod at an altitude of 1272 m above sea level. She observes two ships in the harbour below. The first ship is at S3.3°E, with an angle of depression that measures 6.9°. The second ship is at S15.5°E, with an angle of depression that measures 7.3°. Determine the distance between the two ships, to the nearest metre.

The Eye of the Wind was built in 2009. The power that it generates is about 20% of the total power required for Grouse Mountain. There is an elevator up to the viewing pod, where visitors can see Vancouver and the surrounding mountains.

Rae's Solution

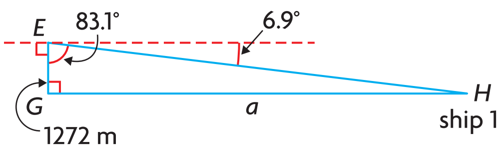


I sketched a 3-D diagram of this situation. I noticed that there are two right triangles.

I decided to draw the right triangles separately.

Let a represent the horizontal distance from Rae to ship 1.

Let b represent the horizontal distance from Rae to ship 2.



The angle between the altitude of the viewing platform and the horizontal measures 90°. If the angle of depression measures 6.9°, then the measure of the complementary angle in the triangle is 83.1° because these measures must add to 90°.

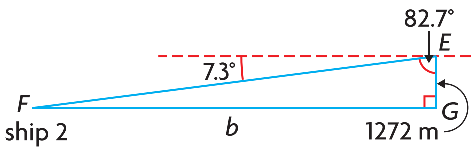
In $\triangle EGH$:

$$\tan 83.1° = \frac{a}{1272}$$

$$1272 \tan 83.1° = \left(\frac{a}{1272}\right)1272$$

$$10\ 511.2416... = a$$

These are right triangles, so I used the tangent ratio to determine the horizontal distance from the base of the mountain, a and b, to each ship.

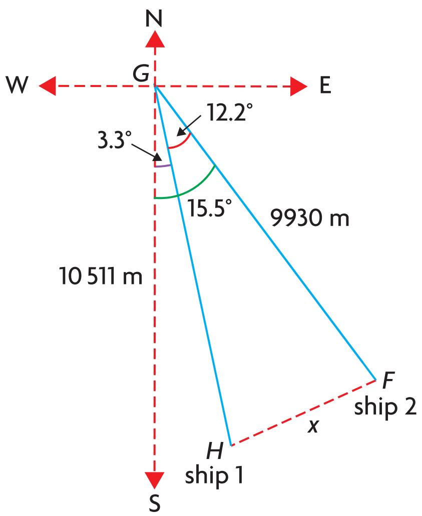


In $\triangle EGF$:

$$\tan 82.7° = \frac{b}{1272}$$

$$1272 \tan 82.7° = \left(\frac{b}{1272}\right)1272$$

$$9929.5133... = b$$



I drew the situation, as seen from above the wind turbine.

Both compass directions are measured against south, so I drew a north–south line and the approximate sight lines to each ship.

To determine the measure of the angle between the two sight lines, I subtracted:

15.5° − 3.3° = 12.2°

$$x^2 = (9930)^2 + (10\,511)^2 - 2(9930)(10\,511)\cos 12.2°$$
$$x^2 = 5\,052\,701.96$$
$$x = 2247.8216...$$

I noticed that I had two known sides and a contained angle, so I used the cosine law to determine the distance between the two ships.

The distance between the two ships is about 2248 m.

This distance appears appropriate, according to my diagram.

Your Turn

If you were on the bridge of ship 2, in what direction would ship 1 be?

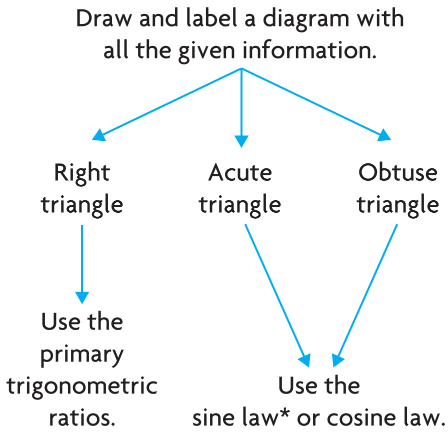
In Summary

Key Idea

- The sine law, the cosine law, the primary trigonometric ratios, and the sum of the measures of the angles in a triangle may all be useful when solving problems that can be modelled using obtuse triangles.

Need to Know

- When solving problems that involve trigonometry, the following decision tree may be useful for choosing an appropriate strategy.

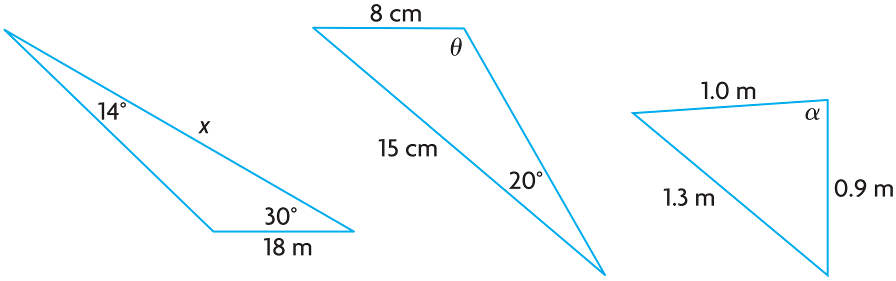


* When you know the lengths of two sides and the measure of an angle that is not contained by the two sides, the case may be ambiguous.

CHECK Your Understanding

1. a) Explain how you would determine the indicated side length or angle measure in each triangle.

i) **ii)** **iii)**

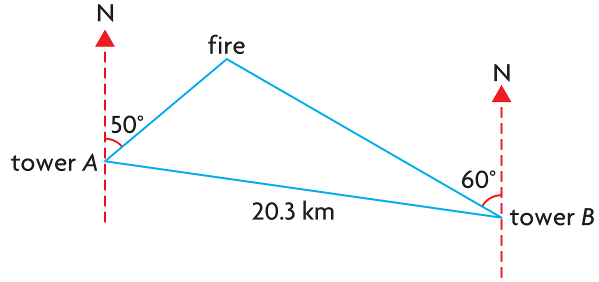


b) Use the strategies you described to determine the indicated side lengths and angle measure in part a). Round your answers to the nearest tenth of a unit.

c) Compare your strategies with a classmate's strategies. Which strategy seems to be more efficient for each triangle?

PRACTISING

2. Two forest-fire towers, A and B, are 20.3 km apart. From tower A, the compass heading for tower B is S80°E. The ranger in each tower sees the same forest fire. The heading of the fire from tower A is N50°E. The heading of the fire from tower B is N60°W. How far, to the nearest tenth of a kilometre, is the fire from each tower?

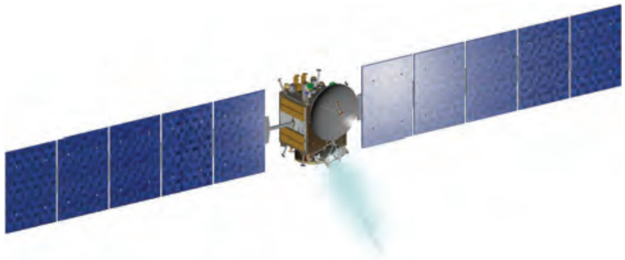


3. The Leaning Tower of Pisa is 55.9 m tall and leans 5.5° from the vertical. What is the distance from the top of the tower to the tip of its shadow, when its shadow is 90.0 m long? (Assume that the ground around the tower is level.) Round your answer to the nearest metre.

4. Shannon wants to build a regular pentagonal sun deck. She is going to use five 2-by-6s, each 12 ft long, to frame the perimeter. She plans to finish the deck with 4 in. cedar planks, laid side by side and parallel to one of the sides. Determine the length of the longest cedar plank.

5. Bijan is hiking in Manning Park, British Columbia. He is hiking alone, but he has a walkie-talkie so that he can keep in touch with his friends at the camp. The walkie-talkies have a range of 6 km. Bijan hikes 5 km along the Skagit Bluffs Trail in a S60°E direction. He then hikes 2 km along the Hope Pass Trail in a N30°E direction.
   - **a)** Draw a diagram to show Bijan's hiking route. Estimate his distance from the camp. Is he still in the range to communicate with his friends at the camp?
   - **b)** Calculate Bijan's distance from the camp. Can he still communicate with his friends at the camp? Explain.

6. On February 28, 2010, Earth was equidistant from the spacecraft *Dawn* and the Sun, forming an isosceles triangle. The distance from Earth to *Dawn* and Earth to the Sun was 0.99 AU (astronomical units). The distance from *Dawn* to the Sun was 1.84 AU.
   a) Draw a diagram to show *Dawn*, Earth, and the Sun.
   b) Determine the angle between the sight lines from Earth to *Dawn* and the Sun.

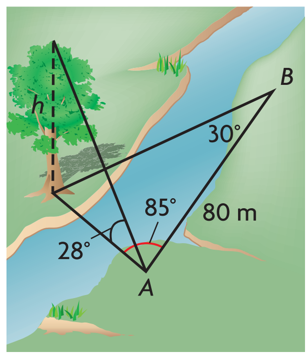


Dawn was launched by NASA on September 27, 2007, with the goal of investigating two of the largest objects in the main asteroid belt: Vesta and Ceres. *Dawn* was to arrive at Vesta in July 2011 and at Ceres in February 2015.

7. A surveyor is measuring the length of a lake. He takes angle measurements from two positions, *A* and *B*, that are 136 m apart and on the same side of the lake. From *B*, the measure of the angle between the sight lines to the ends of the lake is 130°, and the measure of the angle between the sight lines to *A* and one end of the lake is 120°. From *A*, the measure of the angle between the sight lines to the ends of the lake is 65°, and the measure of the angle between the sight lines to *B* and the same end of the lake is 20°. Calculate the length of the lake, to the nearest metre.

8. From an airplane, the angles of depression to two forest fires measure 18° and 35°. One fire is on a heading of N15°W. The other fire is on a heading of S70°E. The airplane is flying at an altitude of 3000 ft. What is the distance between the two fires, to the nearest foot?

9. Bert wants to calculate the height of a tree on the opposite bank of a river. To do this, he lays out a baseline that is 80 m long and measures the angles shown in the diagram. Is the information that Bert has gathered sufficient to determine the height of the tree? Justify your answer.



10. Two towns, Smith Falls and Chester, are 20 km apart. From Smith Falls, the direction to Chester is N70°E. A grass fire has been reported on a bearing of N30°E from Smith Falls and N12°E from Chester. Which town's fire department is closer to the fire? How much closer is it, to the nearest kilometre?

11. Mount Logan, in Yukon Territory, is Canada's highest peak. In North America, it is second in height only to Mount McKinley. An amateur climber is trying to calculate the height of Mount Logan. From her campsite, the angle of elevation to the summit measures 35°. She walks 500 m closer, up a 10° inclined slope, and measures the new angle of elevation as 38°. Her campsite is at an altitude of 1834 m. Determine the height of Mount Logan, to the nearest 10 m.

12. Brit and Tara are standing 8.8 m apart on a dock when they observe a sailboat moving parallel to the dock. When the sailboat is equidistant from both girls, the angle of elevation to the top of its 8.0 m mast is 51° for both girls. Describe how you would determine the measure of the angle, to the nearest degree, between Tara and the boat, as viewed from Brit's position. Justify your answer.

13. A zip line is going to be suspended between two trees. From the forest floor, 12 m from the base of the smaller tree, the angles of elevation to the tree platforms measure 33° and 35°. The distance between the two trees is 35 m.

a) Draw a diagram to represent this situation. What assumptions did you make?

b) Calculate the length of the zip line needed.

14. Determine the angle of depression for the zip line in question 13.

Closing

15. Sketch an obtuse oblique triangle, and label any three measurements (side lengths or angles). Exchange triangles with a classmate. Solve your classmate's triangle, if possible. If your classmate's triangle is impossible to solve, explain to your classmate why it is impossible to solve.

Extending

16. A sailor, out on a lake, sees two lighthouses that are 11 km apart. Lighthouse A is in the direction N47°W and lighthouse B is in the direction N5°W. As seen from lighthouse A, lighthouse B is in the direction N8°E.

a) How far, to the nearest kilometre, is the sailor from each lighthouse?

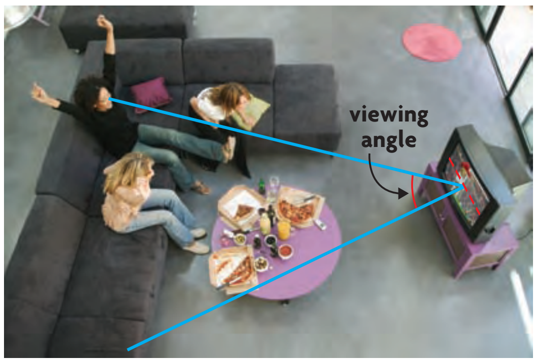
b) Assuming that the shore runs on a straight line through both lighthouses, what is the least distance from the sailor to the shore? Round your answer to the nearest kilometre.

17. An airport radar operator locates two airplanes that are flying toward the airport. The first airplane, P, is 120 km from the airport, A, in a N70°E direction and at an altitude of 2.7 km. The other airplane, Q, is 180 km away, in a S40°W direction and at an altitude of 1.8 km. Calculate the distance between the two airplanes to the nearest tenth of a kilometre.

Math in Action

Measuring the Viewing Angle of a Screen

Television sets, computer monitors, and other screens are often ranked according to their viewing angle. If a screen has a viewing angle of 80°, this means that the colour, brightness, and image quality do not appear to degrade until you are viewing the screen at an angle of 80° or more, measured from the central axis of the screen.



- With a partner or in a small group, make a plan to measure the viewing angle of a screen.
- Test your plan. What adjustments did you need to make as you measured?
- Compare your results with the results of other pairs or groups. Are you satisfied that your plan worked well? Explain.