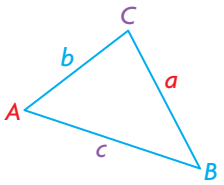
4.1 Exploring the Primary Trigonometric Ratios of Obtuse Angles

YOU WILL NEED

- calculator

GOAL

Determine the relationships between the primary trigonometric ratios of acute and obtuse angles.



sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine law

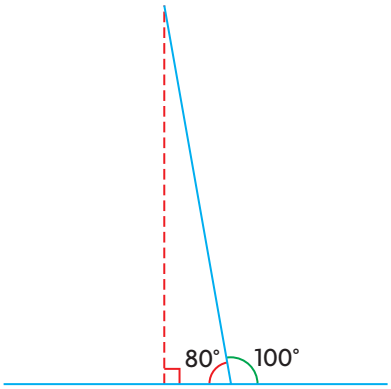
$$a^2 = b^2 + c^2 - 2bc \cos A$$

oblique triangle

A triangle that does not contain a 90° angle.

EXPLORE the Math

Until now, you have used the primary trigonometric ratios only with **acute angles**. For example, you have used these ratios to determine the side lengths and angle measures in right triangles, and you have used the sine and cosine laws to determine the side lengths and angle measures in acute **oblique triangles**.



Joe investigated the values of the primary trigonometric ratios for **obtuse angles**. Using a calculator, he determined that the value of sin 100° is 0.9848… .

He knew that he could not create a right triangle with a 100° angle. However, he knew that he could create a triangle using the supplement of 100°, which is 80°. Out of curiosity, he evaluated sin 80° and determined that it has the same value, 0.9848… .

Joe decided to broaden his investigation. He created a table like the one below.

θ	sin θ	cos θ	tan θ	(180° − θ)	sin (180° − θ)	cos (180° − θ)	tan (180° − θ)
100°	0.9848			80°	0.9848		
110°							
120°							
130°							
180°							

? What relationships do you observe when comparing the trigonometric ratios for obtuse angles with the trigonometric ratios for the related supplementary acute angles?

Reflecting

A. Compare your observations with a classmate's observations. How are they different? How are they alike?

B. Describe any patterns you observed as the measure of the obtuse angle increased.

In Summary

Key Idea

- There are relationships between the value of a primary trigonometric ratio for an acute angle and the value of the same primary trigonometric ratio for the supplement of the acute angle.

Need to Know

- For any angle θ,
  $\sin\theta = \sin(180° - \theta)$
  $\cos\theta = -\cos(180° - \theta)$
  $\tan\theta = -\tan(180° - \theta)$

FURTHER Your Understanding

1. Which of the following equations are valid? Give reasons for your choices.

a) $\sin 25° = \sin 65°$ **d)** $\sin 122° = \sin 58°$

b) $\cos 70° = -\cos 110°$ **e)** $\cos 135° = \cos 45°$

c) $\tan 46° = \tan 134°$ **f)** $\tan 175° = -\tan 5°$

2. Calculate each ratio to four decimal places. Predict another angle that will have an equal or opposite trigonometric ratio. Check your prediction.

a) $\sin 15°$ **c)** $\tan 35°$

b) $\cos 62°$ **d)** $\sin 170°$

3. Determine two angles between 0° and 180° that have each sine ratio.

a) 0.64 **c)** 0.95

b) $\frac{1}{3}$ **d)** $\frac{7}{23}$

4. a) Identify pairs of angles with equal sine ratios in the five triangles to the right.

b) What do you know about the cosine and tangent ratios for these pairs of angles?