3.1 Exploring Side–Angle Relationships in Acute Triangles

YOU WILL NEED

- dynamic geometry software OR ruler and protractor

Inukshuks can have many meanings. Some inukshuks direct travel, some indicate rich fishing or hunting areas, and some warn of danger.

GOAL

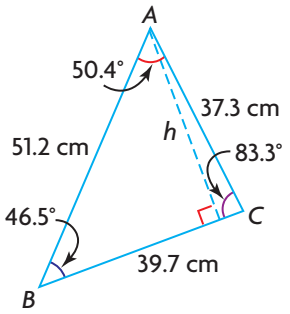
Explore the relationship between each side in an acute triangle and the sine of its opposite angle.

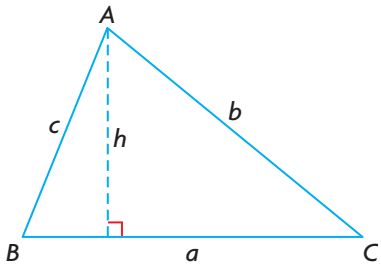
EXPLORE the Math

As they explore the North, the Inuit leave stone cairns, called inukshuks, as markers for those who follow in their path.

You have used the primary trigonometric ratios to determine side lengths and angle measures in right triangles. Can you use primary trigonometric ratios to determine unknown sides and angles in all acute triangles?

Choose one of the triangles below. The first triangle is a scale diagram of the side of the inukshuk shown. The second triangle represents a general acute triangle.





? What are two equivalent expressions that represent the height of $\triangle ABC$?

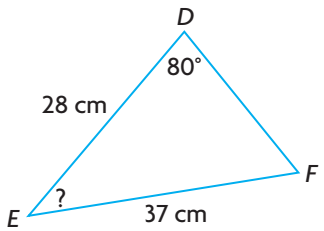
Reflecting

A. Find a classmate who chose a different triangle than you did. Compare each set of expressions. How are they the same and how are they different?

B. If you drew the height of $\triangle ABC$ from a different vertex, how would the expressions for that height be different? Explain.

C. Create an equation using the expressions you created in part A. Show how your equation can be written so that each ratio in the equation involves a side and an angle. Repeat for the expressions you described in part B.

D. Explain how you could determine the measure of $\angle E$ in this acute triangle.



In Summary

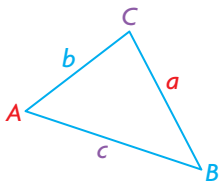
Key Idea

- The ratios of $\frac{\text{length of opposite side}}{\sin(\text{angle})}$ are equivalent for all three side–angle pairs in an acute triangle.

Need to Know

- In an acute triangle, $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



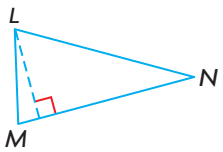
FURTHER Your Understanding

1. For each acute triangle,

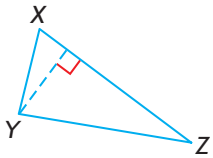
i) copy the triangle and label the sides.

ii) write two expressions for the height of each triangle, and use your expressions to create equivalent ratios.

a)



b)



2. **i)** Sketch a triangle that corresponds to each equation below.

ii) Solve for the unknown side length or angle measure. Round your answer to one decimal place.

a) $\frac{w}{\sin 50°} = \frac{8.0}{\sin 60°}$

b) $\frac{k}{\sin 43°} = \frac{9.5}{\sin 85°}$

c) $\frac{6.0}{\sin M} = \frac{10.0}{\sin 72°}$

d) $\frac{12.5}{\sin Y} = \frac{14.0}{\sin 88°}$

3. Michel claims that if x and y are sides in an acute triangle, then:

$$x \sin Y = y \sin X$$

Do you agree or disagree? Justify your decision.

4. If you want to determine an unknown side length or angle measure in an acute triangle, what is the minimum information that you must have?

5. Do you think the ratios of $\frac{\text{opposite side}}{\sin(\text{angle})}$ are equivalent for all three side–angle pairs in a right triangle? Construct two right triangles, and measure their sides and angles. Use your measurements to test your conjecture.

Communication Tip

The expression $x \sin Y$ is a product. It is equivalent to $x(\sin Y)$.