8.5 Similar Objects: Scale Models and Scale Diagrams

GOAL

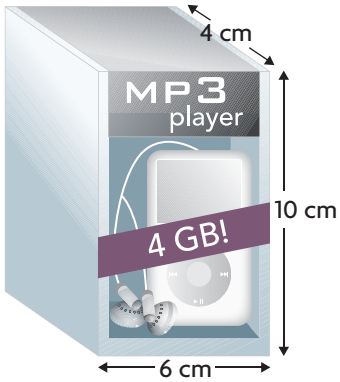
Understand and use scale models and scale diagrams that involve 3-D objects.

YOU WILL NEED

- calculator
- ruler
- centimetre grid paper
- scissors
- tape

INVESTIGATE the Math

Sameer is an engineer for an electronics company. His company currently sells a popular mini-MP3 player. It comes packaged in a box with the dimensions shown.



His team of engineers has designed a larger version of the mini-MP3 player, which has improved features and greater storage capacity. A new box must be created for this new MP3 player.

? How can you create a new box that is similar to the original box?

A. Make a net for the original box.

B. Choose a reasonable scale factor that you can apply to the net you made in part A in order to create a larger box.

C. Make the new net for the larger box.

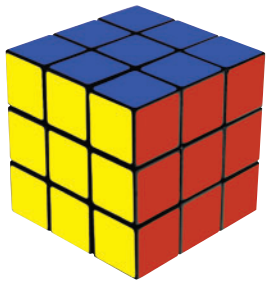
D. Use your net to construct a model of the larger box.

E. How are the dimensions of the boxes related?

F. Are your boxes **similar objects**? Explain.

EXPLORE...

- How do the dimensions of each small cube relate to the overall dimensions of the Rubik's Cube®?



similar objects

Two or more 3-D objects that have proportional dimensions.

Reflecting

G Are the lengths of the diagonals of each pair of corresponding faces in both boxes proportional?

H. Compare the new box you created to the new boxes created by your classmates. Are all of these boxes similar? Explain.

I. Juan claims that other kinds of objects, such as triangular prisms, cylinders, pyramids, cones, and spheres, can be similar, provided that they have the same shape. Do you agree or disagree? Explain.

J. Is it possible for two irregular 3-D objects to be similar? Explain.

APPLY the Math

EXAMPLE 1 Determining if two objects are similar

Sandeep is a chef. In his restaurant, he uses frying pans of various sizes. Are his frying pans similar?

Sandeep's Solution

$$\frac{\text{Bottom diameter of large pan}}{\text{Bottom diameter of small pan}} = \frac{30\text{ cm}}{20\text{ cm}} \text{ or } \frac{3}{2}$$

They look similar. To check, I measured corresponding parts of the pans and compared my measurements.

$$\frac{\text{Depth of large pan}}{\text{Depth of small pan}} = \frac{6\text{ cm}}{4\text{ cm}} \text{ or } \frac{3}{2}$$

$$\frac{\text{Handle length of large pan}}{\text{Handle length of small pan}} = \frac{24\text{ cm}}{16\text{ cm}} \text{ or } \frac{3}{2}$$

The two pans are similar objects. The large pan is an enlarged version of the small pan, by a factor of 1.5.

The corresponding measurements of the pans are proportional.

Your Turn

The top diameter of the large pan is 33 cm. Determine the top diameter of the small pan.

EXAMPLE 2 Determining actual dimensions from a scale model

Esmerelda bought this toy tractor to give to her younger brother for his birthday. The dimensions of the toy are given in the diagram to the right. The scale ratio on the package is 1:16. She knows that her brother will want to know the size of the real tractor. How can she determine the dimensions of the real tractor?

Esmerelda's Solution

The scale model is similar to the real tractor.

I know that all scale models are similar to the real objects. Their measurements are proportional to the corresponding measurements of the real objects.

The scale factor is $\frac{1}{16}$.

I need to multiply each of the dimensions of the model by 16.

Since the model is a reduction of the real tractor, the real tractor is an enlargement of the model by a scale factor of 16.

Actual height $= 16(12.7 \text{ cm})$
Actual height $= 203.2 \text{ cm}$

Actual width $= 16(9.5 \text{ cm})$
Actual width $= 152.0 \text{ cm}$

Actual length $= 16(19.1 \text{ cm})$
Actual length $= 305.6 \text{ cm}$

Actual height $= 2.032 \text{ m}$
Actual width $= 1.520 \text{ m}$
Actual length $= 3.056 \text{ m}$

I converted the measurements from centimetres to metres by multiplying by $\frac{1 \text{ m}}{100 \text{ cm}}$. This made the numbers more meaningful for my brother.

The actual height of the real tractor is about 2.0 m, the actual width is about 1.5 m, and the actual length is about 3.1 m.

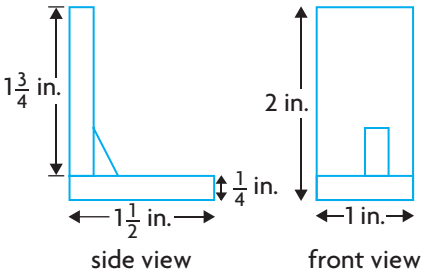
Your Turn

The diameter of the rear tires on the model is 6.0 cm. What is the diameter of the rear tires on the real tractor?

EXAMPLE 3 Enlarging from a scale diagram to determine actual dimensions

Nadia has found plans for a bookend in a woodworking magazine. The plans include a scale diagram, with a scale ratio of 1:5.

Determine the dimensions (length, width, height, and base thickness) of the actual bookend.



Nadia's Solution

3-D scale diagrams are similar to the real objects.

I know that all 3-D scale diagrams are similar to the real objects. Their measurements are proportional to the corresponding measurements of the real objects.

The scale factor is

$$\text{Diagram} : \text{Actual} = 1:5 \text{ or } \frac{\text{Diagram}}{\text{Actual}} = \frac{1}{5}$$

To determine the measurements of the actual bookend from the scale diagram, I need to determine the reciprocal of the scale factor given. Since $\frac{5}{1}$ is greater than 1, the actual bookend is an enlargement of the scale diagram.

$$\text{Basel ength} = \left(1\frac{1}{2}\text{ in.}\right)5$$

I multiplied each dimension in the diagram by 5 to determine the actual dimensions of the bookend.

$$\text{Base length} = \frac{15}{2}\text{ in. or } 7\frac{1}{2}\text{ in.}$$

$$\text{Baset hickness} = \left(\frac{1}{4}\text{ in.}\right)5$$

$$\text{Base thickness} = \frac{5}{4}\text{ in. or } 1\frac{1}{4}\text{ in.}$$

$$\text{Basew idth} = (1\text{ in.})5$$

$$\text{Base width} = 5\text{ in.}$$

$$\text{Overallh eight} = \left(1\frac{3}{4}\text{ in.} + \frac{1}{4}\text{ in.}\right)5$$

$$\text{Overall height} = (2\text{ in.})5 \text{ or } 10\text{ in.}$$

The dimensions of the actual bookend are $7\frac{1}{2}$ in. by 5 in. by 10 in., with a base thickness of $1\frac{1}{4}$ in.

Your Turn

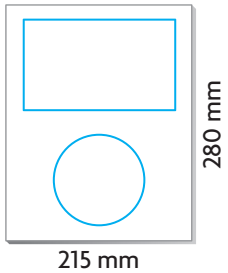
What would the dimensions of the bookend have been if the scale ratio on the plans had been 2:9?

EXAMPLE 4 Drawing a scale diagram of a 3-D object

Céline is an engineer. She is working on a city project, replacing old storm-sewer pipes with new concrete pipes. Each pipe has an inner diameter of 1.50 m, a wall thickness of 0.18 m, and a length of 2.5 m. How can she create a scale drawing of one of these pipes?

Céline's Solution

Drawing the entire pipe, as I see it, involves perspective. This will distort the actual measurements. Drawing a side view and a front view of the pipe will enable me to use proportional measurements.



I decided to split my piece of paper in half. I will use the top half to draw a scale diagram of the side view of the pipe, and the bottom half to draw a scale diagram of the front view.

Paper width:Actual length of pipe $= 215 \text{ mm}:2500 \text{ mm}$
The ideal scale ratio for the width of the paper is about 1:12.

I compared the width of the paper to the length of the pipe, in millimetres.

Paper length:2(Actual diameter of pipe) $= 280 \text{ mm}:3500 \text{ mm}$
The ideal scale ratio for the length of the paper is also about 1:12.

To check that the diagram would fit on the length of the paper, I compared the length of the paper to the diameter of two pipes, with allowance for a space between them. I estimated that this would be about 3.5 m before scaling.

Using a scale factor of $\frac{1}{20}$:

I decided to round down to $\frac{1}{20}$. This scale factor is less than $\frac{1}{12}$, and 20 is a number that is easy to divide into the actual measurements, resulting in numbers I can draw line segments for accurately.

Scale diagram pipe length $= \frac{2.5 \text{ m}}{20}$ or 0.125 m

Scale diagram inner diameter $= \frac{1.5 \text{ m}}{20}$ or 0.075 m

Scale diagram wall thickness $= \frac{0.18 \text{ m}}{20}$ or 0.009 m

I divided each of the actual measurements by 20 to determine the corresponding measurements on the scale diagram.

Scale diagram pipe length $= 125 \text{ mm}$
Scale diagram inner diameter $= 75 \text{ mm}$
Scale diagram wall thickness $= 9 \text{ mm}$

Since the smallest unit on my ruler is mm, I multiplied each measure by $\frac{1000 \text{ mm}}{1 \text{ m}}$ to convert the measurements to millimetres.

Side view:

Length = 125 mm

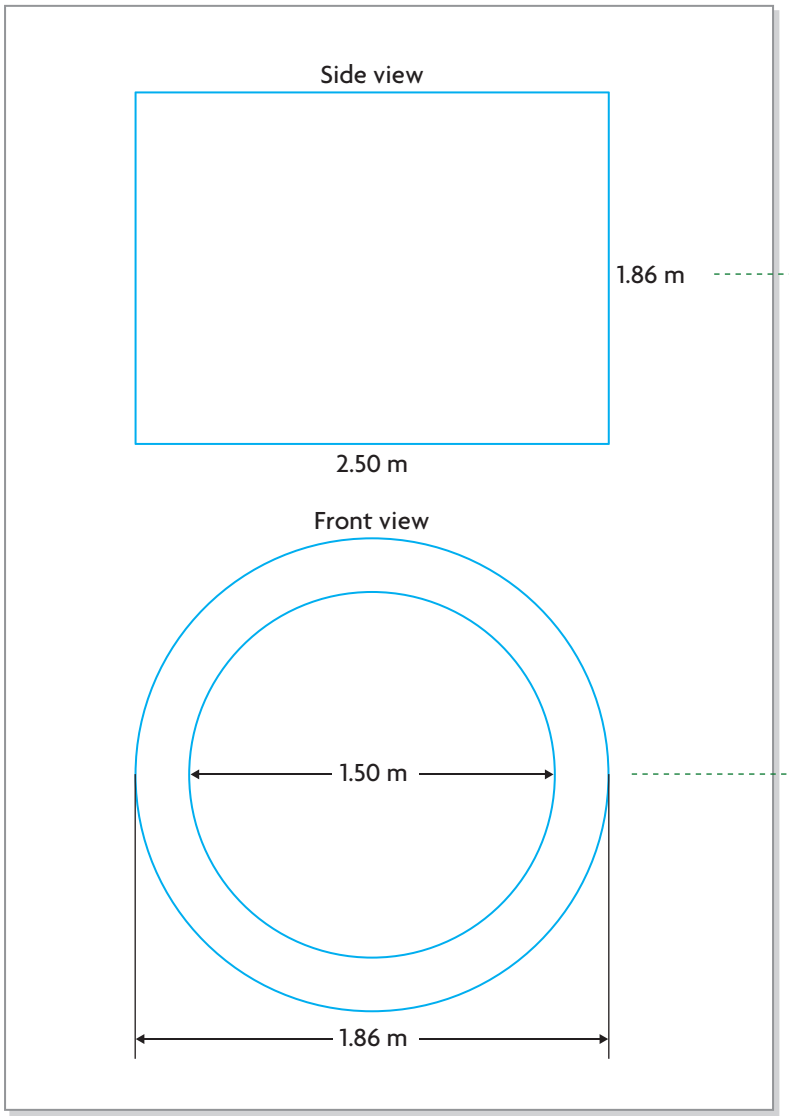
Width = 75 mm + 9 mm + 9 mm or 93 mm

The width of the pipe is the sum of the inner diameter and twice the thickness of its wall.

Front view:

Inner diameter = 75 mm

Outer diameter = 93 mm



The side view of the pipe will look like a rectangle. I added the inner diameter to the wall thickness at the top and bottom of the pipe to determine the width of the pipe.

The front view looks like two circles. The outer diameter corresponds to the width of the rectangle.

Shown at 50% of actual size

Your Turn

Draw a scale drawing of the pipe using a scale factor of $\frac{1}{15}$.

In Summary

Key Ideas

- Two 3-D objects that are similar have dimensions that are proportional.
- The scale factor is the ratio of a linear measurement of an object to the corresponding linear measurement in a similar object, where both measurements are expressed using the same units.
- To create a scale model or diagram, determine an appropriate scale to use based on the dimensions of the original object and the size of the model or diagram that is required.

Need to Know

- You can multiply any linear measurement of an object by the scale factor to calculate the corresponding measurement of the similar object.
- You can determine the scale factor k, used to create a scale model of an object by using any corresponding linear measurements of the object and the scale model:

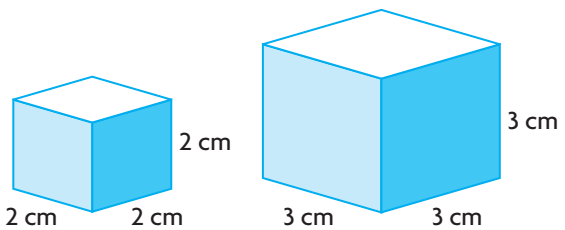
$$k = \frac{\text{Linear measurement of scale model}}{\text{Corresponding linear measurement of object}}$$

- When a scale factor is between 0 and 1, the new object is a reduction of the original object.
- When a scale factor is greater than 1, the new object is an enlargement of the original object.

CHECK Your Understanding

1. For each of the following, determine whether the two objects are similar and justify your decision.

a)



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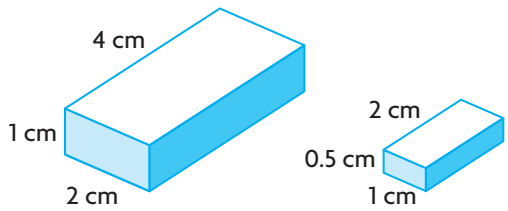

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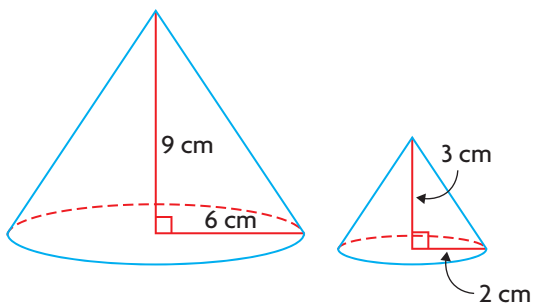

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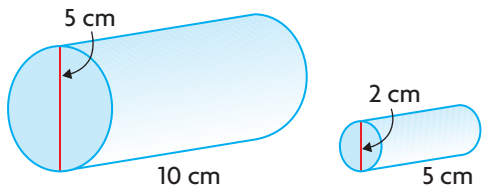
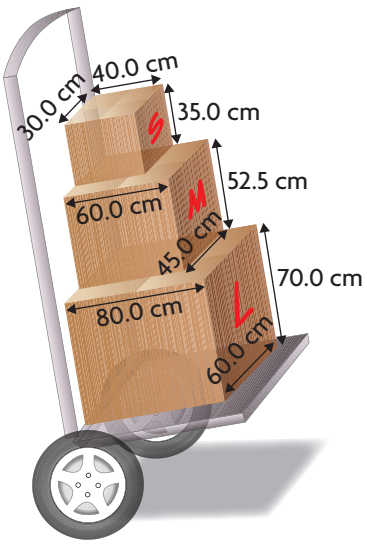


2. The National Basketball Association (NBA) uses a basketball with a diameter of 25 cm. The Women's National Basketball Association (WNBA) uses a basketball with a diameter of 22 cm.
   a) Are these balls similar? Explain.
   b) Determine the scale factor that relates
      i) the NBA ball to the WNBA ball
      ii) the WNBA ball to the NBA ball

3. One of the most famous ships in Canadian sailing history is the *Bluenose*. Launched in 1921 from Nova Scotia as a fishing vessel, the *Bluenose* operated in the rough waters off the coast of Newfoundland. The *Bluenose* became very famous for its speed, winning all the great classic sailing races on the American east coast. Mark has a 1:100 scale model of this ship. The model has a length of 52 cm, a beam (width) of 8.5 cm, and a height of 43 cm. Calculate the length, beam, and height of the actual ship.

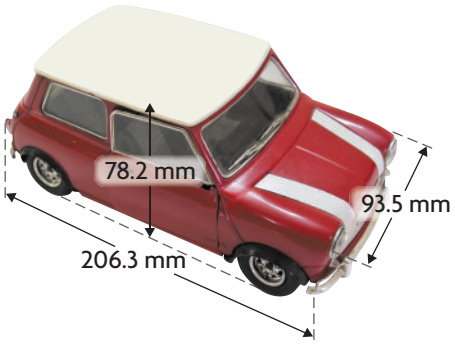
PRACTISING



4. Toni works for a moving company. The company sells three different-sized boxes, as shown.
   a) Are the boxes similar? Explain.
   b) The letters on the boxes (S, M, L for small, medium, large) increase in height in proportion to the size of the box. The red M on the medium box is 24 cm tall. Determine the heights of the S and the L.

5. Last summer, Ed visited the Royal Tyrrell Museum in Drumheller, Alberta, to see the fossil and dinosaur exhibits. While he was there, he purchased a 1:40 scale model of the *Albertosaurus libratus*, which was native to the area over 70 million years ago. The length of the model is 21.5 cm, and the height is 9.5 cm. Determine the length and height of this species of dinosaur.

6. A 1:18 scale model of a car has the dimensions shown. Determine the dimensions of the actual car.



7. The bald eagle is Canada's largest bird of prey. It has a body length of about 90 cm and, while perched, a height of about 75 cm. Hank is a woodcarver who wants to create a carving, to scale, of a bald eagle while perched. He has a block of wood that is 150 cm long, 150 cm wide, and 200 cm high.

a) Suggest a scale factor that Hank could use for his carving.

b) If he uses the scale factor you suggested in part a), determine the height and length of the eagle he will create.

8. A carving of Tecumseh, the Shawnee leader of a confederacy that fought in the war of 1812, is located in the Wood Carving Museum in Windsor, Ontario. The carving is $6\frac{1}{2}$ ft tall by $2\frac{1}{2}$ ft wide. The museum wants to sell replica models that are 26 in. tall in the gift shop.

a) What scale factor must be used to produce these models?

b) Determine the width of these models.

9. Umiaks are boats that are used in the Arctic for transportation and for traditional whale hunting. The frame of an umiak is built from spruce wood. Traditionally, the outer cover was made from animal skins, such as walrus and bearded-seal skins, but today it can be made from ballistic nylon. A typical umiak is 32 ft long, with a beam (width) of 48 in. Determine these dimensions on a scale model built using a scale ratio of 1:24.

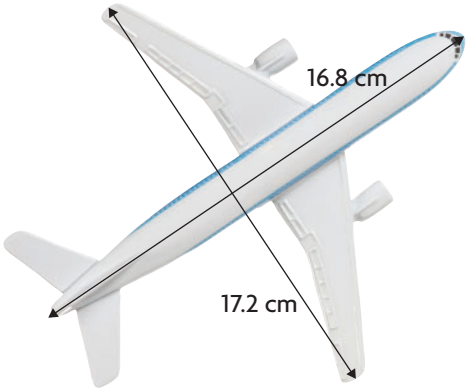
10. Some model train enthusiasts enjoy building villages on their layouts. Two popular scale ratios for model trains are HO (1:87) and N (1:160). Nick has found a building that he would like to add to his N-scale layout, but its dimensions are for an HO-scale layout. The dimensions are 6 in. long by $8\frac{1}{2}$ in. tall by 4 in. wide.

a) Estimate what the dimensions of the building would be in N scale.

b) Determine the conversion ratio for HO:N.

c) Determine the dimensions of the building in N scale, to the nearest eighth of an inch.

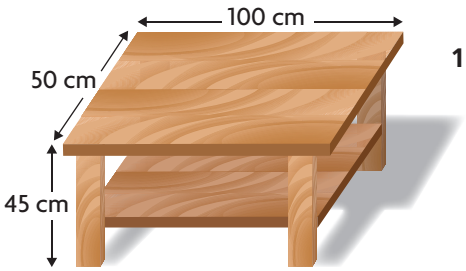
11. The measurements of a scale model of a passenger jet are shown. The model was made using a scale factor of $\frac{1}{200}$. The floor of Hangar 77 at the Calgary International Airport measures 46.6 m long by 71.9 m wide. How many of these passenger jets could fit in this hangar?



12. Take a photograph of a structure or building, as well as a referent—an object with a known height or length. For example, you could include a metre stick or another student in your photograph.

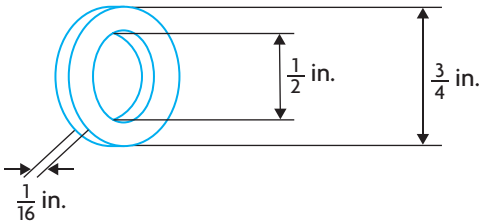
a) Estimate the measurements of the structure or building using only your photograph.

b) Describe how you used a scale factor to determine the measurements.



13. Draw a scale diagram that shows the top, front, and side views of this coffee table. Assume a uniform thickness of 5 cm for all the pieces of wood. Each leg is inset 10 cm from the edge of the tabletop, and the bottom shelf is 10 cm above the ground. Use a scale factor of $\frac{1}{10}$.

14. The specifications for a steel washer are shown.



Draw a scale diagram of the top, side, and front views of the washer, using a scale ratio of 4:1.

15. This chest freezer has the dimensions indicated. Draw a scale diagram of the top, side, and front views of the freezer, so that all three views fit on a single page of standard paper.

16. Choose an object in your classroom, and measure its dimensions. Draw a scale diagram of the object that shows top, side, and front views.

17. Suppose that you increase the dimensions of a box, which is the shape of a rectangular right prism, by 150%.

a) Do you think the area of the base of the box will also increase by this scale factor? Justify your decision.

b) Will the volume of the box also increase by this scale factor? Justify your decision.

Closing

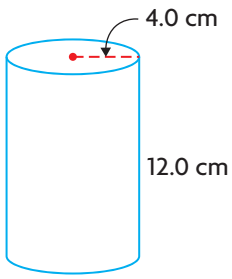
18. How is the process for solving problems that involve similar objects the same as the process for solving problems that involve similar shapes? How is the process different?

Extending

19. A juice company plans to enlarge this can by a scale factor of 1.5.

a) The new can will be made from the same metal, in the same thickness, as the smaller can. By what factor will the cost of the metal increase?

b) The cost of the metal that is needed to make the larger can is \$0.045. Determine the cost of the metal that is needed to make the smaller can.



20. The surface area of a right cone is 100 cm^2. Its dimensions are reduced by 50% to produce a similar cone. Determine the surface area of the similar cone.