8.3 Scale Diagrams

YOU WILL NEED

- calculator
- grid paper
- ruler
- protractor

EXPLORE...

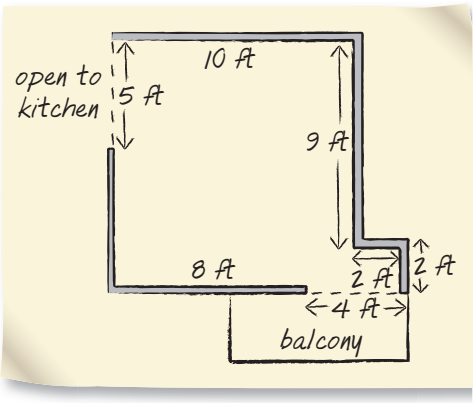
- Construct a scale drawing that models an airplane flying N20°E at 160 km/h for 2.5 h. What factors must you keep in mind?

GOAL

Understand and use scale diagrams involving 2-D shapes.

INVESTIGATE the Math

Maxine is moving into a new apartment. Before moving day, she wants to decide where to place her furniture in her new living room. When she visited the apartment, she drew this rough sketch of the room's layout and recorded some measurements.



She has also measured her large furniture, which she wants placed by the movers. These measurements are shown in the table below.

Furniture	Dimensions (width by length)
couch	40 in. by 90 in.
loveseat	40 in. by 66 in.
wall unit	20 in. by 60 in.

scale diagram

A drawing in which measurements are proportionally reduced or enlarged from actual measurements; a scale diagram is similar to the original.

scale

The ratio of a measurement on a diagram to the corresponding distance measured on the shape or object represented by the diagram.

? How can you use a **scale diagram** of this room, on an 8.5 in. by 11 in. sheet of paper, to determine where to place these pieces of furniture?

A. Determine a **scale** you can use to create a scale diagram of the living room on an 8.5 in. by 11 in. sheet of paper.

B. Use your scale to determine what the lengths of walls and openings in your scale diagram should be.

C. Create your scale diagram of the living room.

D. Use your scale to determine the dimensions of each piece of furniture that needs to be placed.

E. Select a strategy to determine a good location for each piece of furniture. Add the three pieces of furniture to your scale diagram.

Reflecting

F. Compare your diagram with your classmates' diagrams. How are they the same, and how are they different?

G. Maxine used a **scale factor** of $\frac{1}{16}$ to create her diagram. Explain the advantages of using this scale factor.

H. Does it make sense that the scale factor you used for your diagram had to be less than 1? Explain.

scale factor

A number created from the ratio of any two corresponding measurements of two similar shapes or objects, written as a fraction, a decimal, or a percent.

Communication *Tip*

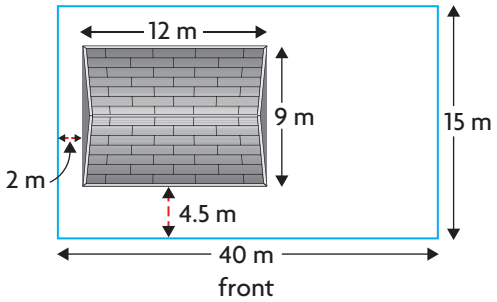
A scale is a ratio or rate, so it always includes units. A scale factor is a number without units.

APPLY the Math

EXAMPLE 1 Drawing a 2-D scale diagram that requires a reduction

A builder plans to construct a house on a rectangular lot, as shown in this sketch.

Draw a scale diagram of the lot and house using a scale of 1 m : 500 m.



Eric's Solution

Lot dimensions:

$\text{Lot length} = 40\text{ m}\left(\frac{1}{500}\right)$

$\text{Lot length} = 0.08\text{ m}$

$\text{Lot width} = 15\text{ m}\left(\frac{1}{500}\right)$

$\text{Lot width} = 0.03\text{ m}$

House dimensions:

$\text{House length} = 12\text{ m}\left(\frac{1}{500}\right)$

$\text{House length} = 0.024\text{ m}$

$\text{House width} = 9\text{ m}\left(\frac{1}{500}\right)$

$\text{House width} = 0.018\text{ m}$

Since the scale is 1 m : 500 m, which is less than 1, I knew that my scale diagram would be a reduction of the actual house and lot. The scale factor is $\frac{1\ \cancel{m}}{500\ \cancel{m}} = \frac{1}{500}$. I multiplied all of the measurements by the scale factor.

Insets from left and front of lot:

$\text{Front inset} = 4.5\text{ m}\left(\frac{1}{500}\right)$

$\text{Front inset} = 0.009\text{ m}$

$\text{Left inset} = 2\text{ m}\left(\frac{1}{500}\right)$

$\text{Left inset} = 0.004\text{ m}$

Lot dimensions:

$\text{Lot length} = 0.08 \text{ m}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$

$\text{Lot length} = 8.0 \text{ cm}$

$\text{Lot width} = 0.03 \text{ m}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$

$\text{Lot width} = 3.0 \text{ cm}$

House dimensions:

$\text{House length} = 0.024 \text{ m}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$

$\text{House length} = 2.4 \text{ cm}$

$\text{House width} = 0.018 \text{ m}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$

$\text{House width} = 1.8 \text{ cm}$

All the measurements I calculated for my scale diagram are in metres, but my ruler is in centimetres. To draw the diagram accurately, I converted the measurements to centimetres.

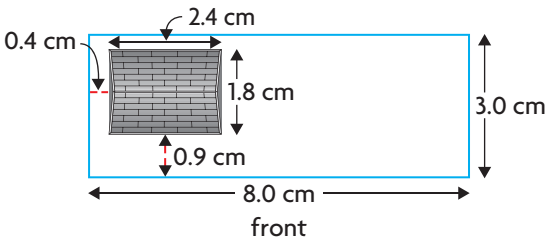
Insets from left and front of lot:

$\text{Front inset} = 0.009 \text{ m}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$

$\text{Front inset} = 0.9 \text{ cm}$

$\text{Left inset} = 0.004 \text{ m}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$

$\text{Left inset} = 0.4 \text{ cm}$



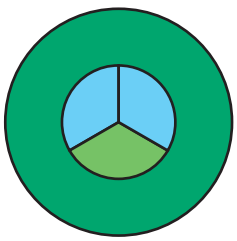
I used the measurements I calculated to draw the lot and then the house. I noticed that I could draw the rectangle for the house only one way, since the house had to be inside the lot by a margin of 4.5 m on one side.

Your Turn

Joe decided to draw a scale diagram of the lot and house using a scale factor of 0.01. Explain how his diagram would differ from Eric's diagram.

EXAMPLE 2 Drawing a 2-D scale diagram that requires an enlargement

Jess designed the logo shown for an environment club. She wants to enlarge the logo so that it can be applied to the front of a baseball cap. The hat company has suggested a scale factor of $\frac{5}{3}$. Draw a scale diagram of the logo as it will appear on the baseball cap.



Jess's Solution

Diameter of outer circle = 3 cm
Diameter of inner circle = 1.5 cm
Length of each line segment = 0.75 cm
Measure of all sector angles = 120°

I measured the diameters of the outer and inner circles on my logo design, and I recorded the measurements. I thought that the three line segments were radii of the inner circle, but I measured them just to be safe. I also measured the angle of each sector.

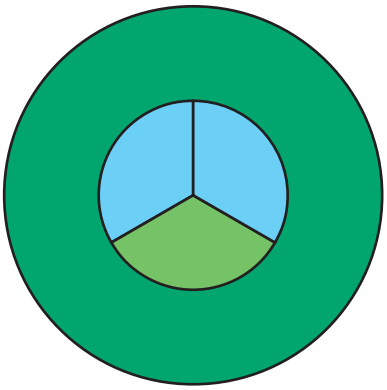
New outer diameter = $3 \text{ cm}\left(\frac{5}{3}\right)$ or 5 cm

New inner diameter = $1.5 \text{ cm}\left(\frac{5}{3}\right)$ or 2.5 cm

New line segments = $0.75 \text{ cm}\left(\frac{5}{3}\right)$ or 1.25 cm

Measure of all new sector angles = 120°

I calculated the new measurements for the enlarged logo by multiplying each linear measurement by the scale factor of $\frac{5}{3}$. Since the original logo and the enlarged logo are similar, I knew the measure of all the sector angles would be 120°.



I drew the larger circle by setting my compass radius to 5 cm. Then I drew the smaller circle by using the same centre and setting my compass radius to 2.5 cm. I drew the vertical radius of the inner circle and used a protractor to measure angles of 120° from this radius to draw the other two radii. Finally, I coloured in the enlarged logo using the same colours I used for my logo design.

Your Turn

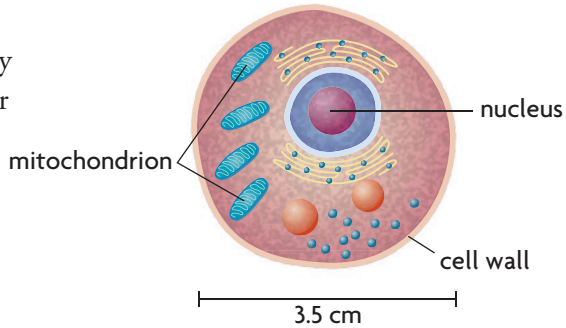
Jess initially thought of using a scale factor of 400%.

a) Draw the logo using this scale factor.

b) Why do you think she decided to use the recommended scale factor?

EXAMPLE 3 Determining scale factor

The diameter of the animal cell that is represented by this scale diagram is actually 0.25 mm. What scale factor was used to draw this scale diagram?



Hannah's Solution

Let k be the scale factor of the diagram.

$$k = \frac{\text{Diagram measurement}}{\text{Actual measurement}}$$

$$k = \frac{35 \text{ mm}}{0.25 \text{ mm}}$$

The diameter of the cell in the diagram is 3.5 cm, and the actual diameter is 0.25 mm. I expressed both measurements in millimetres and then wrote a ratio.

$$k = \frac{35}{0.25}$$

$$k = \frac{3500}{25}$$

To eliminate the decimal, I multiplied both terms in the ratio by 100 and then simplified.

$$k = \frac{140}{1}$$

The scale factor used for the scale diagram is 140.

Communication *Tip*

The variable k is often used to represent an unknown scale factor.

Your Turn

Explain why it makes sense that the scale factor used for this scale diagram is greater than 1.

In Summary

Key Ideas

- Scale diagrams can be used to represent 2-D shapes. To create a scale diagram, you must determine an appropriate scale to use. This depends on the dimensions of the original shape and the size of diagram that is required.
- The scale factor represents the ratio of a distance measurement of a shape to the corresponding distance measurement of a similar shape, where both measurements are expressed using the same units.

Need to Know

- You can multiply any linear dimension of a shape by the scale factor to calculate the corresponding dimension of a similar shape.
- When determining the scale factor, k, used for a scale diagram, the measurement from the original shape is placed in the denominator:

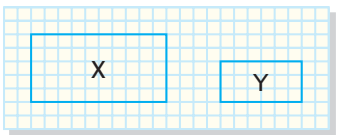
$$k = \frac{\text{Diagram measurement}}{\text{Actual measurement}}$$

- When a scale factor is between 0 and 1, the new shape is a reduction of the original shape.
- When a scale factor is greater than 1, the new shape is an enlargement of the original shape.

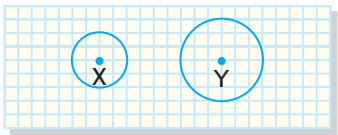
CHECK Your Understanding

1. Determine the scale factor that was used to transform diagram X into diagram Y. Express your scale factor as a fraction and as a percent.

a)



b)



2. Determine if the original will be larger or smaller than the scale diagram after the given scale factor is applied.

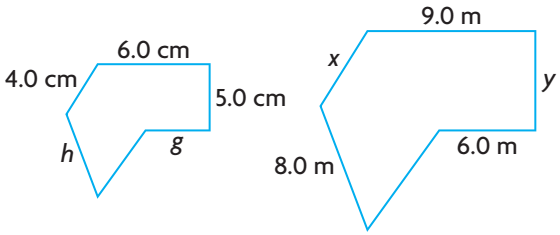
a) scale factor: 112% **b)** scale factor: 0.75 **c)** scale factor: $\frac{4}{9}$

3. On a plan, an actual length of 6 ft is represented by 5 in.

a) Determine the scale of the plan.

b) Determine the scale factor used to make the plan.

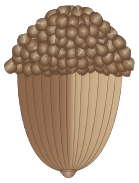
4. The following two polygons are similar. Determine the lengths of sides g, h, x, and y to the nearest tenth of a unit.



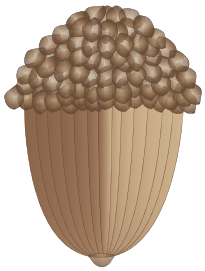
PRACTISING

5. The Garry oak is a tree that is found on Vancouver Island. The original acorn for these scale diagrams was 1.9 cm long. Determine the scale factor that was used for each diagram.

a)



b)



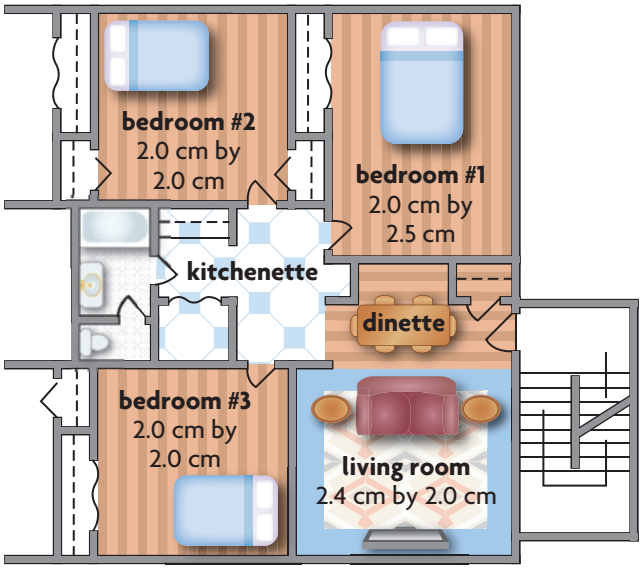
c)

6. The floor plan of an apartment is shown to the right, drawn using a scale factor of 0.005.

a) What are the actual dimensions of each bedroom?

b) What are the actual dimensions of the living room?

c) Which room has the greatest area?

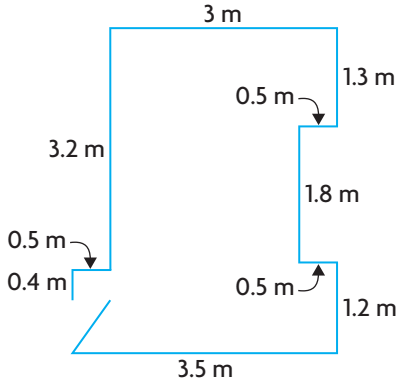


7. Yani wants to make a scale diagram of the floor plan of his school. He wants his diagram to fit on an 8.5 in. by 11 in. sheet of paper. The school is 650 ft long and 300 ft wide at its widest point.

a) What would be a reasonable scale for Yani to use so that his diagram will fit on the sheet of paper?

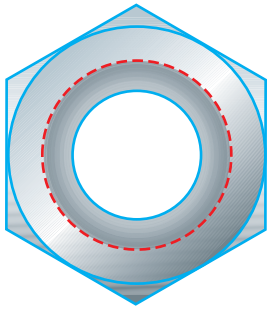
b) Assume that the school's floor plan is a rectangle. Draw a scale diagram using the scale you determined in part a).

8. Ken has made a sketch of the floor plan of his bedroom. Draw an accurate scale diagram of his bedroom on 1 cm grid paper, using a scale factor of $\frac{1}{50}$.



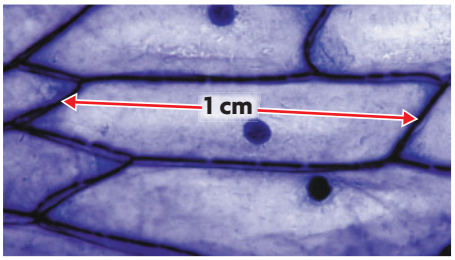
9. A computer chip on a circuit board has a rectangular shape, with a width of 6 mm and a length of 9 mm. Plans for the circuit board must be drawn using a scale factor of 15. Draw a scale diagram of the computer chip as it would appear on the plans.

10. This top view of a hex-nut must be enlarged by a scale factor of 250% for a display at a trade show.



- **a)** Measure the necessary distances on the diagram.
- **b)** Determine what the corresponding distances on the enlarged diagram should be.
- **c)** Draw the scale diagram of the hex-nut.

11. Sara has a microscope with a lens that magnifies by a factor of 40. She was able to capture the image of a slide containing onion cells, as shown. In the image, the cell was about 1 cm long. How long is the actual onion cell, to nearest hundredth of a millimetre?

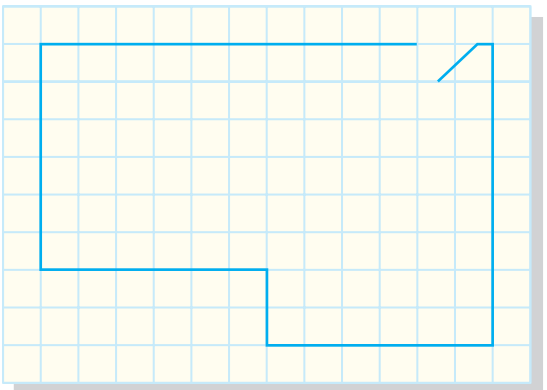


12. **a)** Using the map scale, estimate the distance from

- **i)** Yellowknife to Fort Norman
- **ii)** Fort Providence to Fort Norman

b) Of the three locations on the map, which two are closest to each other?

13. This scale diagram, drawn on 0.5 cm grid paper, shows the floor plan of a greenhouse, drawn using a scale ratio of 1 : 75.



- **a)** Determine the perimeter of the greenhouse.
- **b)** Determine the area of the floor of the greenhouse, to the nearest tenth of a square metre.

14. Determine the scale factor used in each situation.

a) The actual diameter of a penny is 19 mm. In a scale diagram, the diameter of a penny is 5.7 cm.

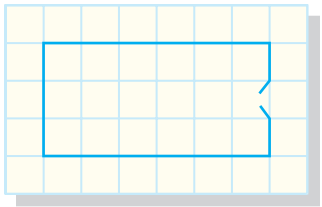
b) The actual width of a door is 30 in. In a scale diagram, the width of the door is $1\frac{1}{2}$ in.

c) The diagonal of an actual stamp is 2.5 cm long. In a scale diagram, the diagonal is 1.0 m long.

d) The height of an actual communications tower is 55 ft. In a scale diagram, the height of the tower is 6 in.

15. A billboard measures 4.5 m by 3.5 m. Draw a scale diagram of the billboard that fits in a space measuring 20 cm by 15 cm.

16. The floor plan for a garden shed is shown below. The area of the actual floor is 72 m^2.



a) Determine the actual area that each square on the floor plan represents.

b) Determine the actual distance that 5 mm on the floor plan represents.

c) Determine the scale of the plan.

d) Determine the scale factor that was used to draw the floor plan.

17. The viewing areas of most LCD televisions are similar rectangles. Regardless of the size of a television, the length : width ratio is often 16 : 9. Rahj has built-in bookshelves that are 4 ft wide. There is a vertical distance of 26 in. between each shelf. Show that a 42 in. LCD television will fit on one of these shelves.

Communication *Tip*

The given measurement for a television is the length of a diagonal across the viewing area.

18. **a)** Use geometric shapes to create a logo that will fit in a space measuring 12 cm by 12 cm.

b) Draw a scale diagram of your logo using a scale factor of 25%.

c) Draw a scale diagram of your logo using a scale factor of 1.5.

Closing

19. When drawing a scale diagram on a sheet of paper, how do you decide what scale factor to use? What do you need to consider?

Extending

20. Sanjay has 34 in. of red oak moulding that is 1 in. wide. He would like to use this moulding to frame a photograph, as shown. The photograph measures 12 in. by 8 in., so the frame would require more moulding than he has.

a) By what scale factor should he reduce the photograph so that he can use the wood he has to make the frame?

b) What are the dimensions of the reduced photograph?