8.2 Solving Problems That Involve Rates

YOU WILL NEED

- calculator
- graph paper
- ruler

EXPLORE...

- A car travels at 80 km/h. What other rates, expressed using different units, could be used to describe the speed of the car? What would be some of the advantages of using these other rates?

GOAL

Analyze and solve problems that involve rates.

LEARN ABOUT the Math

Jeff lives in a town near the Canada–U.S. border. The gas tank of his truck holds about 90 L. He can either buy gas in his town at $1.06/L or travel across the border into the United States to fill up at $2.86 U.S./gal.

? Which option makes the most sense economically?

EXAMPLE 1 Solving a problem that involves multiple rates

Jeff's Solution

The cost to fill up at a gas station in Canada, D, is

$$D = (90\ \cancel{L})\left(\frac{\$1.06\ \text{Cdn}}{1\ \cancel{L}}\right)$$
$$D = \$95.40\ \text{Cdn}$$

It will cost $95.40 to fill up in Canada.

First, I determined the cost to fill up at a gas station in Canada.

Converting 90.0 L into U.S. gallons, G, is

$$G = (90\ \cancel{L})\left(\frac{1\ \text{gal}}{3.79\cancel{L}}\right)$$
$$G = 23.746...\ \text{gal}$$

I needed to convert the volume that my gas tank holds in litres into U.S. gallons. I know that 1 U.S. gallon is equivalent to 3.79 L.

The cost in U.S. dollars, U, for about 23.7 U.S. gal is

$$U = (23.746...\ \cancel{\text{gal}})\left(\frac{\$2.86\ \text{U.S.}}{1\ \cancel{\text{gal}}}\right)$$
$$U = \$67.915...\ \text{U.S.}$$

The cost in Canadian dollars, C, for about 23.7 U.S. gal is

$$C = (\$67.915...\ \cancel{\text{U.S.}})\left(\frac{\$1.02\ \text{Cdn}}{1\ \cancel{\text{U.S.}}}\right)$$
$$C = \$69.273...\ \text{Cdn}$$

I determined the cost to fill up in U.S. dollars. I needed to convert U.S. dollars into Canadian dollars. The exchange rate today is $1 U.S./$1.02 Cdn.

It will cost $69.27 Cdn to fill up in the United States.

Difference in cost = $95.40 − $69.273...

Difference in cost = $26.13...

Today, it is more economical for me to fill up in the United States. I will save about $26.

How much I save depends on the price of gas at each station.

Reflecting

A. What other factors will affect Jeff's savings each time he considers where to fill up?

B. Jeff has only considered the cost to fill up his truck. What other factors should he consider when deciding where he will buy gas?

C. Jeff thinks that saving less than $10 is not worth his time. If he had half a tank of gas in his truck, would it be worthwhile for him to fill up in the United States today? Justify your answer.

APPLY the Math

EXAMPLE 2 Connecting rates to contextual situations

Describe a situation in which each unit rate might be used. Identify and explain factors that could influence the unit rate in this situation.

a) 0.05 mg/kg **b)** 98.5¢/L **c)** 7.2 MBps

Mangat's Solution

a) 0.05 mg/kg could be the rate at which a certain type of medicine must be administered. This rate means that 0.05 mg of medication is needed for each kilogram of a patient's mass. The type of medication used could influence the quantity administered per kilogram of body mass.

I needed a situation in which a very small mass of a substance (milligrams) is related to a kilogram. I knew that medicine is prescribed according the mass of a patient, which can be measured in kilograms.

b) 98.5¢/L could represent the rate at which consumers pay for 1 L of gasoline. This rate could be influenced by the type of gas chosen. It could also be influenced by the current cost of gasoline per barrel, which could be affected by war, weather, time of year, holidays, and supply and demand.

I needed a situation in which a cost in cents is related to a volume in litres. Gasoline is sold in litres.

c) 7.2 MBps could be the rate at which information is transferred over a computer network. This rate could be influenced by

- the type of network (wireless versus wired).
- the type/quality of the network card.
- the type of router used.

I needed a situation in which megabytes (MB) are related to time in seconds. Data transfer can be measured this way.

Your Turn

Think of two other unit rates that you are familiar with. State one or two factors that could influence these rates.

EXAMPLE 3 Reasoning to solve a rate problem

Paula is asked to order snacks for an office meeting of 180 people. She decides to order dessert squares, which come in boxes of 24. She estimates that she will need 2.5 squares/person. How many boxes should she buy?

Mila's Solution: Calculating using unit analysis

Formula to describe the snack order:

$$\text{Number of boxes} = \left(\frac{1 \text{ box}}{\text{Number of squares}}\right)\left(\frac{\text{Number of squares eaten}}{1 \text{ person}}\right)(\text{Number of people})$$

$$\text{Number of boxes} = \left(\frac{1 \text{ box}}{24 \cancel{\text{squares}}}\right)\left(\frac{2.5 \cancel{\text{squares}}}{1 \cancel{\text{person}}}\right)(180 \cancel{\text{persons}})$$

Paula estimated that each person would eat about 2.5 squares.

$$\text{Number of boxes} = 18.75$$

Paula should buy 19 boxes.

I rounded up my answer to the nearest number of boxes.

Joe's Solution: Estimating using proportional reasoning

There are about 25 squares in each box. If each person eats 2.5 squares, then Paula needs one box for every 10 people. There are 18 groups of 10 in 180.

I decided to estimate. I knew that each person will eat about 2.5 squares. Estimating 25 squares/box made the numbers easier to work with, using mental math.

Paula needs to buy at least 18 boxes. She should order 19 boxes to be safe.

I know that I underestimated, since I estimated 25 squares/box and there are only 24 squares/box.

Your Turn

If each person at the meeting eats about 1.5 squares on average, how many boxes of squares will be left over?

EXAMPLE 4 Solving a problem that involves different rates

Amelia walks briskly, at 6 km/h. When she walks at this rate for 2 h, she burns 454 Cal. Bruce walks at a slower rate, 4 km/h, burning 62 Cal in 30 min. If Amelia walks for 3 h, how much longer will Bruce have to walk in order to burn the same amount of Calories?

April's Solution: Using a function

The amount of Calories that Amelia burns each hour when she walks at 6 km/h, A, is

$$A = \frac{454 \text{ Cal}}{2 \text{ h}}$$

$$A = 227 \text{ Cal/h}$$

If $A(t)$ represents the amount of Calories that Amelia burns and t represents the time in hours, then

$A(t) = 227t$

I assumed that Amelia walks at a constant rate, so I could use a linear function to represent the relation between Calories burned and time.

For 3 h,

$A(3) = 227(3)$

$A(3) = 681$ Cal

Amelia burns 681 Cal in 3 h.

The amount of Calories that Bruce burns each hour when he walks at 4 km/h, B, is

$$B = \frac{62 \text{ Cal}}{0.5 \text{ h}}$$

$B = 124$ Cal/h

If $B(t)$ represents the amount of Calories that Bruce burns and t represents the time in hours, then

$B(t) = 124t$

I assumed that Bruce also walks at a constant rate.

For 681 Cal,

$B(t) = 681$ Cal

$681\text{Cal} = (124 \text{ Cal/h})t$

$5.491\ldots \text{ h} = t$

I needed to know how long it takes Bruce to burn 681 Cal, so I substituted 681 Cal for B(t).

Bruce will need to walk for about 5.5 h to burn the same amount of Calories that Amelia burns in 3 h. Bruce will need to walk for an additional 2.5 h.

I rounded the time to the nearest tenth of an hour.

Joanna's Solution: Using equivalent ratios

The rate for Calories burned is Cal/h. When Amelia walks at 6 km/h for 2 h, she burns 454 Cal. When she walks for 3 h, she burns x Calories. These rates are equivalent.

I assumed that Amelia walks at a constant rate, so the rate at which she burns calories will also be constant.

$$\frac{454 \text{ Cal}}{2 \text{ h}} = \frac{x}{3 \text{ h}}$$

$$3 \text{ h}\left(\frac{454 \text{ Cal}}{2 \text{ h}}\right) = x$$

$681\text{Cal} = x$

I wrote an equation using equivalent rates to determine the amount, in Calories, that Amelia burns in 3 h. The units in the ratios are the same, so I am confident that my answer will be in the correct units, Calories.

Bruce burns 62 Cal in 0.5 h. He must burn 681 Cal in t hours.

$$\frac{62 \text{ Cal}}{0.5 \text{ h}} = \frac{681 \text{ Cal}}{t}$$

$62t = 340.5$ h

$t = 5.491\ldots$ h

I wrote an equation using a pair of equivalent rates, where 681 Cal must be burned by Bruce to match Amelia. Then I solved for t.

Converting 0.491... h into minutes, M, is

$$M = (0.491...\ \text{h})\left(\frac{60\ \text{min}}{1\ \text{h}}\right)$$

$$M = 29.516...\ \text{min}$$

Bruce will need to walk for about 5 h 30 min to burn the same amount of Calories that Amelia burns in 3 h.

Bruce will need to walk for about 2 h 30 min longer to burn the same amount of Calories.

Your Turn

If Bruce walks for 2 h, for how long does Amelia need to walk to burn the same amount, in Calories, as Bruce? Round your answer to the nearest minute.

In Summary

Key Idea

- When you are given a rate problem that involves an unknown, you can solve the problem using a variety of strategies.

Need to Know

- Often, a problem that involves rates can be solved by writing an equation that involves a pair of equivalent ratios. To be equivalent ratios, the units in the numerators of the two ratios must be the same, and the units in the denominators must be the same. Paying attention to the units in each term of the ratios will help you write the equation correctly.
- A multiplication strategy can be used to solve many rate problems, such as problems that require conversions between units. Including the units with each term in the product and using unit elimination helps you verify that your product is correct.
- When a rate of change is constant, writing a linear function to represent the situation may be useful when solving problems.

CHECK Your Understanding

1. a) 50 L of oil costs $163. How much oil, to the nearest litre, could you buy for $30?

b) It takes 3 min 25 s to fill a 75 L gas tank. How long, to the nearest minute, will it take to fill a 55 L gas tank?

c) 8 kg of beef costs $68.00. How much will it cost, to the nearest cent, for 1.5 kg of beef?

d) The adult dosage of an antibiotic medicine is 25 mL/80 kg. How much medicine is needed for a person with a mass of 95 kg?

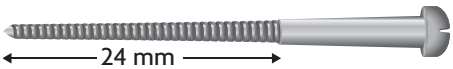
2. Two competing stores have 350 mL cans of pop on sale this week. Supersaver is selling a case of 24 cans for $5.99. Gord the Grocer is selling cans of the same pop in cases of 12, with three cases for $9.99.

a) Which store is selling soft drinks at the lower price per can?

b) Besides price, what other factors should be considered when determining which store offers the better buy for a consumer?

PRACTISING

3. A screw has 32 turns over a distance of 24 mm of thread.



Another screw, with the same pattern, has 42 mm of thread. How many turns does it have?

4. The Wildcats won 12 of their first 20 games. At this rate, predict how many games they will win during the 30-game season.

5. Mario borrowed $1000 and paid $40 simple interest. If he borrowed the money for eight months, what interest rate was he charged?

6. Describe a situation in which each rate might be used. Identify any factors that could influence the rate in this situation.

a) $7.23/kg

b) 20 mL/90 kg

c) $1.08/100 g

d) −1.5 °C/km

e) 20 g/L

f) $4.99/ft^2

7. Basic units of data are transferred by a particular computer at 12 MB (megabytes) every 2 s. How long will it take this computer to transfer 1.5 GB (gigabytes) of data? (1 GB is equivalent to 1024 MB.)

8. Melanie wants to defrost a frozen roast, which weighs 2.68 kg, in her microwave. To find out how much time she needs, she looks in a cookbook. She reads that 2 lb of meat takes 15 min to defrost. How long, to the nearest minute, should she set the timer for?

Communication Tip

1 kg ≐ 2.2 lb

9. A nurse administers a vaccine that comes in a 10 mL bottle. The adult dosage is 0.5 cc (1 cc = 1 mL). How many adults can the nurse vaccinate before the bottle is empty?

10. Tonya works 50 h every three weeks. At this rate, how many hours will she work in one year? Explain how you could solve this problem using two different strategies.

11. Chris and her friend Elena drove from Vancouver to Yellowknife for a reunion. They took turns driving, so they only needed to stop for gas or food. They drove the 2359 km distance in 36 h 12 min.

a) Determine their average speed to the nearest tenth of a kilometre per hour.

b) They used 231.2 L of fuel. Determine their average fuel consumption per 100 km.

c) Chris and Elena spent $252.05 on fuel. What was the average cost of a litre of gas?

12. Manpret has taken a job as a nurse in the community health centre in Tuktoyaktuk, Northwest Territories. She plans to ship her car, furniture, and personal effects to Tuktoyaktuk by barge from Vancouver. She has found these shipping rates online:

- light-duty vehicles: $0.2015/lb
- furniture and personal effects: $0.2734/lb

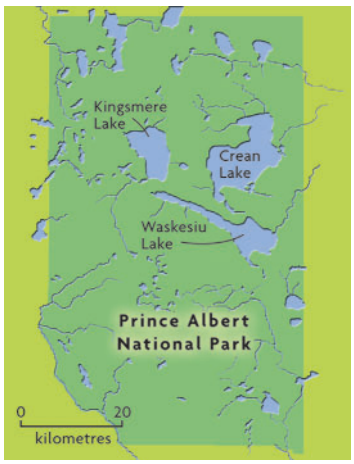
Manpret knows that her car has a mass of 1250 kg. She estimates that she has roughly 550 lb of furniture and personal effects. Calculate her cost to ship these items to her destination.

13. Emma runs a kennel near Wild Horse, Alberta. She has decided to purchase dog food from a U.S. supplier. The supplier sells 40 lb bags for $38.95 U.S. The exchange rate is $1 U.S. for $1.05 Cdn on the day that she orders the food.

a) How much, in Canadian dollars, does Emma spend to buy 20 bags of dog food?

b) Each dog eats about 4 kg/week and Emma boards an average of 12 dogs per day. Will the 20 bags of dog food last two months? Explain.

c) What other factors should she have considered before she ordered from this supplier?



14. The map to the left shows Prince Albert National Park in Saskatchewan. The scale of the map is 1.3 cm to 20 km.

a) Estimate the area of the park in hectares. One hectare (1 ha) is equivalent to 10 000 m^2.

b) The annual cost to monitor and fight forest fires in this region is about $48/ha. Estimate the annual fire management expenditure for the park.

15. Paula wants to buy bottled water.

- Store A, located 12 km from her home, is selling 500 mL bottles in a case of 24 for $4.99.
- Store B, located 20 km from her home, is selling 330 mL bottles in a case of 24 for $3.49.
- It costs Paula $0.14/km to run her car.

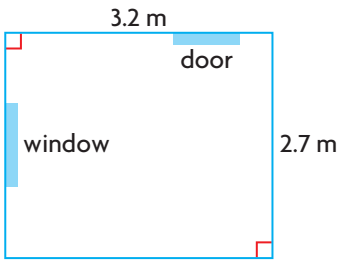
Which store would you recommend for Paula to buy her water? Explain.

16. A cargo jet leaves an airport that is 2000 ft above sea level at 6:30 a.m. The jet climbs steadily to a cruising altitude of 37 000 ft, at a rate of 7000 ft/min. After cruising at this altitude for 40 min, the jet descends steadily at a rate of 3500 ft/min to an airport that is 5500 ft above sea level. What time does the jet land?

17. The low temperature for a certain day was recorded as -5.3 °C at 3:30 a.m. The temperature then rose steadily until the high temperature was recorded as 11.8 °C at 5:45 p.m. A weather forecaster predicted the same temperature increase rate for the next day, from a low of -7 °C at 3 a.m. Estimate the temperature at 7 a.m. the next day.

Closing

18. A particular type of paint can be purchased at two local stores. Bren's Interior Design sells the 870 mL size for $7.99, while Home Suppliers sells the 3.7 L size for $27.99. This type of paint will cover an area of 10 m^2/L. Suppose that you want to paint a room that is 2.4 m high and has the dimensions shown to the right. One wall has a door that measures 80 cm by 205 cm. Another wall has a window that measures 100 cm by 130 cm.



a) Based on cost, at which store should you buy the paint?

b) In addition to cost, what other factors should you consider when deciding where to buy the paint?

Extending

19. A pendulum is pulled to the left. When released, it swings from left to right, but never returns to its initial position. The time required for one complete oscillation is called the *period* of the pendulum. The time, T, in seconds, for one period of the pendulum is given by the equation

$$T = 2\pi\sqrt{\frac{L}{9.8}},$$

where L is the length of the pendulum in metres. How many periods will a 2 m pendulum complete over 1 h?

20. A new saltwater pool is being filled by four different pumps, which pump water from a nearby ocean into the pool. The first pump can fill the entire pool with water in two days. The second pump requires three days, and the third pump requires four days. The fourth pump needs only 6 h. How long will it take to fill the pool if all four pumps are used?

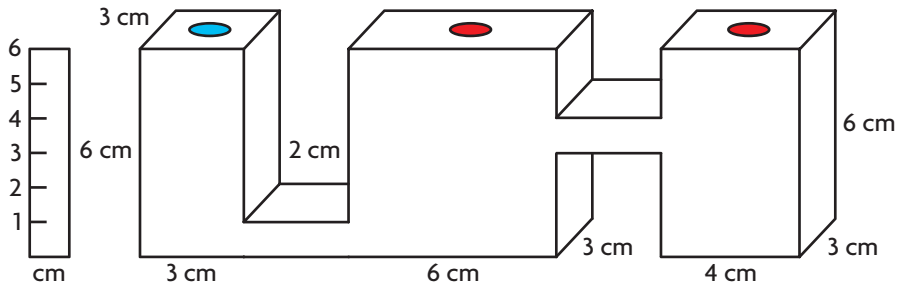
Applying Problem-Solving Strategies

Analyzing a Rate Puzzle

Filling irregular shaped containers with water at a constant rate can produce some interesting results.

The Puzzle

A container is constructed from a connected sequence of rectangular prisms as shown below.



Each connecting piece has dimensions of 1 cm by 2 cm by 3 cm. Water is dripping into the hole on the left (marked in blue) at a constant rate of 1 cm^3/min. The marks to the left of the container measure the height, in centimetres, of the water in the container as it fills.

A. Will one prism be filled before the others? Explain.

B. Determine the time needed for the water to reach each of the height marks indicated to the left of the container.

C. Use your times to plot a graph of water height versus time.

D. Between which two height markers did the water level rise at the slowest rate? Explain how you know.

The Strategy

E. Describe the strategy you used to determine the information needed to create your graph.

Modifying the Puzzle

F. There are other holes in the container, indicated in red. If the container were filled at the same rate through either of those holes, would the length of time needed to fill the container change? Explain.

G. Would your graph change if you filled the container through a different hole? Explain.