Comparing and Interpreting Rates

EXPLORE...

- World-class sprinters can run 100 m in about 9.8 s. If they could run at this rate for a longer period of time, estimate how far they could run in a minute, in an hour, and in a day.

Communication | *Tip*

The time 01:20:34.7, or 80:34.7, can also be written as 1 h 20 min 34.7 s or as 80 min 34.7 s.

GOAL

Represent, interpret, and compare rates.

INVESTIGATE the Math

A triathlon consists of three different races: a 1.5 km swim, a 40 km bike ride, and a 10 km run. The time it takes to "transition" from one race to the next is given as "Trans" in the official records. The setup for the transition area is different in different triathlons.

Simon Whitfield of Victoria, British Columbia, participated in the triathlon in both the 2000 and 2008 Olympic Games. His results for both Olympics are shown below.

2000 Sydney Olympic Games Triathlon: Men

			Swimming		Cycling		Running	
Rank	Athlete	Country	Time	Trans	Time	Trans	Time	Total Time
1	Whitfield	Canada	17:56	0:21	58:54	0:17	30:52	1:48:24.02

2008 Beijing Olympic Games Triathlon: Men

			Swimming		Cycling		Running	
Rank	Athlete	Country	Time	Trans	Time	Trans	Time	Total Time
2	Whitfield	Canada	18:18	0:27	58:56	0:29	30:48	1:48:58.47

? How can you compare Simon's speeds in his two Olympic medal-winning triathlons?

A. Compare Simon's times for each race segment in the two triathlons, ignoring the transition times. Which race segment had the greatest difference in Simon's times?

B. Speed, the ratio of distance to time, is an example of a **rate**. Create a distance versus time graph to compare Simon's swimming speeds in these triathlons. Express time in hours and distance in kilometres.

C. Carmen claims that Simon swam faster in the 2000 Sydney Olympic Games Triathlon than in Beijing in 2008. Explain how the data in the table and in the graph you created in part B support her claim.

D. Determine the slope of each line segment on your distance versus time graph. What do these slopes represent?

E. Do the slopes you determined in part D support Carmen's claim? Explain.

rate

A comparison of two amounts that are measured in different units; for example, keying 240 words/8 min

Communication | *Tip*

The word "per" means "to each" or "for each." It is written in units using a slash (/).

Reflecting

F. Can the slopes of line segments on a graph be used to compare rates? Explain.

G. Are the slopes of the line segments on your distance versus time graph *unit rates* ? Explain.

H. Can the swimming speeds in each triathlon be calculated directly from the information given? Explain.

> **unit rate**
> A rate in which the numerical value of the second term is 1; for example, keying 240 words/8 min expressed as a unit rate is 30 words/min.

APPLY the Math

EXAMPLE 1 | Comparing two rates expressed in different units

Natasha can buy a 12 kg turkey from her local butcher for $42.89. The local supermarket has turkeys advertised in its weekly flyer for $1.49/lb. There are about 2.2 lb in 1 kg. Which store has the lower price?

Natasha's Solution: Comparing using estimation

Butcher:
A 12 kg turkey from the butcher costs about $43.

Supermarket:
The price of a turkey from the supermarket, T, is

$$T = \left(\frac{\$1.50}{1 \, \text{lb}}\right)\left(\frac{2 \, \text{lb}}{1 \, \text{kg}}\right)(12 \, \text{kg})$$

$T = \$36$
A 12 kg turkey from the supermarket costs about $36.

A turkey from the butcher costs $\dfrac{\$43}{12 \, \text{kg}}$, and a turkey from the supermarket costs $\dfrac{\$36}{12 \, \text{kg}}$. So, the price of a 12 kg turkey is about $7 less at the supermarket.

> To make a comparison, I needed to estimate the prices of two turkeys that are the same size, measured in the same units.
>
> I multiplied the price per pound by the conversion rate for pounds to kilograms, which is approximately 2 lb/1 kg, and then I multiplied by 12 kg, the mass of the turkey. This gave me an estimate of the price of a similar turkey from the supermarket.

Dimitri's Solution: Comparing using unit rates

Butcher:
The price per kilogram for a turkey from the butcher, B, is

$$B = \frac{\$42.89}{12 \, \text{kg}}$$

$$B = \frac{\$3.574...}{1 \, \text{kg}}$$

> To make a comparison, I expressed each price as a unit rate, using the same units.
>
> I divided the price of a 12 kg turkey from the butcher by the mass. This gave me the price per kilogram for a turkey from the butcher.

The price per kilogram for a turkey from the butcher is $3.57/kg.

Supermarket:

The price per kilogram for a turkey from the supermarket, S, is

$$S = \left(\frac{\$1.49}{1 \text{ lb}}\right)\left(\frac{2.2 \text{ lb}}{1 \text{ kg}}\right)$$

$$S = \frac{\$3.278}{1 \text{ kg}}$$

> I multiplied the price per pound by the conversion rate for pounds to kilograms, which is approximately 2.2 lb/1 kg. This gave me the price per kilogram for a turkey from the supermarket.

The price per kilogram for a turkey from the supermarket is $3.28/kg.

The price of a turkey is 29¢/kg less at the supermarket.

Your Turn

Describe how you could use a graph to compare the price of a turkey from the butcher and from the supermarket.

Connecting the slope of a line segment to a rate

Describe a scenario that could be represented by this graph. Compare the rates shown, and discuss why the rates may have changed.

Distance vs. Time
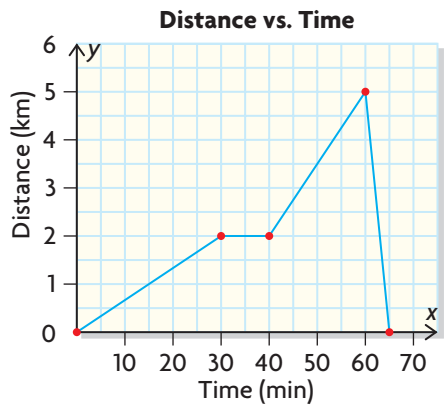
Gilles's Solution

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$$

> To choose a reasonable scenario, I decided to calculate the slope of each line segment. This meant dividing the change in distance by the change in time.

The first line segment has endpoints $(0, 0)$ and $(30, 2)$.

$$\text{Slope}_1 = \frac{2 - 0}{30 - 0}$$

$$\text{Slope}_1 = \frac{2}{30} \text{ km/min}$$

$$\left(\frac{2 \text{ km}}{30 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 4 \text{ km/h}$$

The second line segment is horizontal.

$$\text{Slope}_2 = 0 \text{ km/h}$$

> The slope represents the rate of change of distance over time, in kilometres per minute, for each line segment. This speed is a unit rate, since the numerical value of the second term is equal to 1.

> I converted each rate in kilometres per minute to kilometres per hour to help me describe the scenario. The speed represented by the first line segment is 4 km/h.

The third line segment has endpoints $(40, 2)$ and $(60, 5)$.

$$\text{Slope}_3 = \frac{5 - 2}{60 - 40}$$

$$\text{Slope}_3 = \frac{3}{20} \text{ km/min}$$

$$\left(\frac{3 \text{ km}}{20 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 9 \text{ km/h}$$

The fourth line segment has endpoints $(60, 5)$ and $(65, 0)$.

$$\text{Slope}_4 = \frac{0 - 5}{65 - 60}$$

$$\text{Slope}_4 = \frac{-5}{5} \text{ or } -1 \text{ km/min}$$

$$\left(\frac{-1 \text{ km}}{1 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = -60 \text{ km/h}$$

The graph could represent the distance that a person travelled and the time for the trip. Each line segment on the graph could represent a different part of the trip.

> I decided that the rates of change were close to the rates at which a person could walk, run, and drive. I made up a story to match the graph and the rates of change that I calculated

The first line segment represents a person walking at a rate of 4 km/h for 30 min to get from home to a variety store.

> 4 km/h is a reasonable rate at which a person could walk. The graph shows that the distance is increasing during this period.

The second line segment represents the person stopping at the store for 10 min to buy something or talk to someone.

> This makes sense, since the distance travelled did not change between 30 min and 40 min.

The third line segment represents the person jogging at a rate of 9 km/h for 20 min to get to a friend's house.

> It's difficult to walk at 9 km/h, so I decided that the person must be jogging. The distance was increasing faster during this period.

The fourth line segment represents the person travelling at the greatest rate, 60 km/h, for an additional 5 min. This is because the person received a phone call from home, to say that dinner is ready. The person gets a ride home from a friend.

> The rate of change is negative. This means that the person's distance, relative to the starting point, is decreasing. The person must be travelling back to the starting position. A person can't walk or run at 60 km/h, so I decided that the person must be travelling by car.

EXAMPLE 3 | Solving a problem involving rates

When making a decision about buying a vehicle, fuel efficiency is often an important factor.

The gas tank of Mario's new car has a capacity of 55 L. The owner's manual claims that the fuel efficiency of Mario's car is 7.6 L/100 km on the highway. Before Mario's first big highway trip, he set his trip meter to 0 km so he could keep track of the total distance he drove. He started with the gas tank full. Each time he stopped to fill up the tank, he recorded the distance he had driven and the amount of gas he purchased.

Fill-up	Total Distance Driven (km)	Quantity of Gas Purchased (L)
1	645	48.0
2	1037	32.1

On which leg of Mario's trip was his fuel efficiency the best?

Katrina's Solution: Using a graph

Distance driven on leg 1 = 645 km

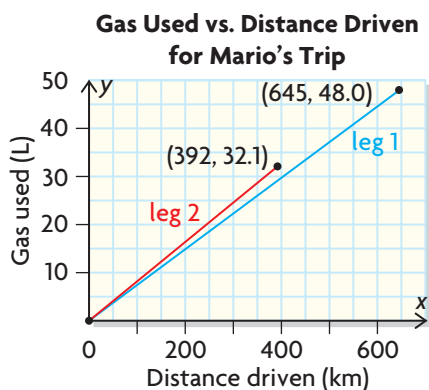Distance driven on leg 2 = 1037 − 645
Distance driven on leg 2 = 392 km

> First, I needed to determine how far Mario drove on each leg of the trip. Since the trip meter was initially set to 0 km, I knew that he drove 645 km on the first leg. I had to subtract the first trip meter reading from the second trip meter reading to determine the distance he drove on the second leg.

Gas used on leg 1 = 48.0 L
Gas used on leg 2 = 32.1 L

> Since Mario began each leg with a full tank, the amount of gas he purchased at each fill-up tells me the amount of gas he used on each leg of the trip.

Gas Used vs. Distance Driven for Mario's Trip

The blue line segment is less steep than the red line segment, so its slope is less. This means the fuel efficiency of the car was better on the first leg of the trip.

I created a graph of Gas used versus Distance driven by plotting the ordered pairs (Distance driven, Gas used). I used distance driven on the horizontal axis, because I noticed that distance is the second term in the manufacturer's fuel efficiency. The slope of the line segment for each leg of the trip represents the change in fuel use over distance.

The fuel efficiency is better when you use less gas to drive the same distance, so I knew I was looking for the line segment that was less steep.

Grant's Solution: Using unit rates

Leg	Distance Driven (km)	Gas Used (L)
1	645 − 0 = 645	48.0
2	1037 − 645 = 392	32.1

I used a table to organize the information I needed to determine the fuel efficiency on each leg of the trip as a unit rate, in litres per kilometre. For each leg of the trip, I had to determine the distance that Mario drove and the amount of gas that was used.

First leg:

$$\text{Fuel efficiency} = \frac{\text{Gas used}}{\text{Distance driven}}$$

$$\text{Fuel efficiency} = \frac{48.0 \text{ L}}{645 \text{ km}}$$

$$\text{Fuel efficiency} = 0.074... \text{ L/km}$$

I calculated the car's fuel efficiency on each leg of the trip by dividing the gas used by the distance driven.

Second leg:

$$\text{Fuel efficiency} = \frac{\text{Gas used}}{\text{Distance driven}}$$

$$\text{Fuel efficiency} = \frac{32.1 \text{ L}}{392 \text{ km}}$$

$$\text{Fuel efficiency} = 0.081... \text{ L/km}$$

The fuel efficiency of the car was better on the first leg of the trip.

The fuel efficiency is better when you use less gas to drive the same distance.

Your Turn

Did the car achieve the manufacturer's fuel efficiency rating of $\dfrac{7.6 \text{ L}}{100 \text{ km}}$ on either leg of the trip? Explain.

8.1 Comparing and Interpreting Rates   **457**
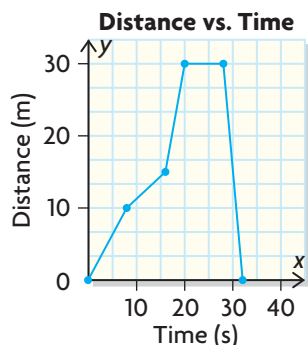
In Summary

Key Idea

- Rates can be represented in a variety of ways. The representation you choose should depend on your purpose.

Need to Know

- You can compare rates by writing them as
  - rates with the same units, with second terms that are numerically the same.
  - unit rates, where the numerical values of the second terms are equal to 1.
- When comparing rates, it is helpful to round the values. This enables you to do mental math and express each rate as an approximate unit rate.
- In a graph that shows the relationship between two quantities, the slope of a line segment represents the average rate of change for these quantities.
- The slope of a line segment that represents a rate of change is a unit rate.

CHECK Your Understanding

1. Compare the following situations, and determine the lower rate.
   a) At store A, 8 kg of cheddar cheese costs $68.
      At store B, 12 kg of cheddar cheese costs $88.20.
   b) At gas station A, 44 L of fuel costs $41.36.
      At gas station B, 32 L of fuel costs $31.36.

2. Compare the following situations, and determine the greater rate.
   a) It takes 4 h 15 min to drain tank A, which holds 300 L of water.
      It takes 2 h 10 min to drain tank B, which holds 150 L of water.
   b) Person A runs 400 m in 1 min 15 s.
      Person B runs 1 km in 5 min 20 s.

Distance vs. Time

3. The graph to the left shows how an all-terrain vehicle (ATV) travels over time.
   a) Over which interval of time is the ATV travelling the slowest? Over which interval is it travelling the fastest?
   b) When does the ATV start to return to its starting position? When does it get there?
   c) What does a zero slope mean in the context of this graph?

PRACTISING

4. Apple juice is sold in 1 L bottles and 200 mL boxes. A 1 L bottle sells for $1.75, and fifteen 200 mL boxes sell for $4.99.
   a) Determine the unit rate, in dollars per millilitre, for each size.
   b) Which size has the lower cost per millilitre?

5. The list price for a 925 mL container of paint is $20.09. A 3.54 L container of the same paint costs $52.99. Which container has the lower unit cost?

6. When Rupi goes to her aerobics class, she can burn 140 Cal in 20 min. When she plays hockey for 1.5 h, she can burn 720 Cal. Which activity burns Calories at a greater rate?

7. For each of the following, compare the two rates and determine the lower rate.
   a) whole chickens: $3.61/kg or 10 lb for $17.40
   b) jogging speeds: 6 mph or 2 km in 10 min
   c) fuel efficiency: 10.6 L/100 km or 35.1 L of fuel needed to travel 450 km
   d) driving speeds: 30 m/s or 100 km/h

8. Jay can buy a 25 lb bag of bird seed for $21.30 from the Farmers Co-op. The pet store in town sells an 18 kg bag for $24.69. At which store can Jay buy bird seed at a lower cost? Explain how you know.

9. Shelley has two choices for a long-distance telephone plan:
   • her telephone company, which charges 4¢/min
   • a device that plugs into her Internet modem, which costs $19.95 with an additional charge of 1.5¢/min
   Shelley makes, on average, 50 min of long-distance calls per month. Which option would be cheaper on an annual basis? Justify your decision.

10. On Monday, a crew paved 10 km of road in 8 h. On Tuesday, the crew paved 15 km in 10 h. Draw a graph to compare the crew's daily paving rates.

11. Draw a graph that shows how, over one day, the outdoor temperature starts at 24 °C, decreases at a rate of 1.5 °C/h for 5 h, remains constant for 2 h, and then increases by 0.75 °C/h for 4 h.

12. A hotel shuttle bus takes David from the airport to his hotel. Use the distance versus time graph to the right to create a story that describes David's bus trip.

Courtesy Bus Trip
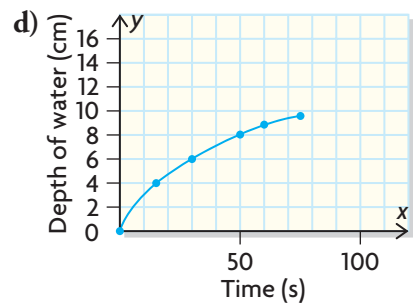
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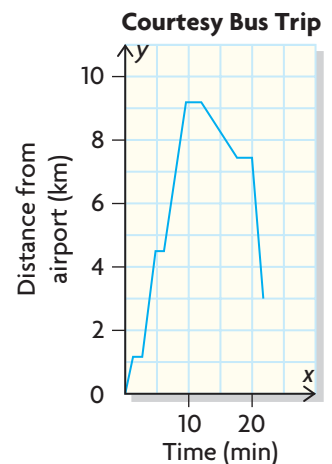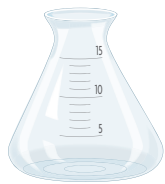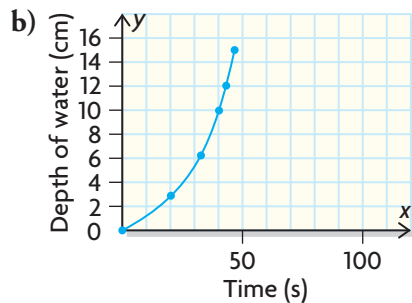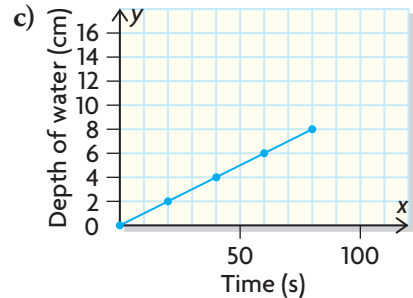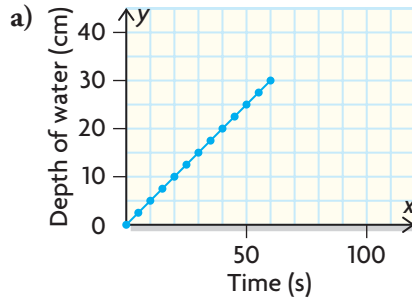
flask

beaker
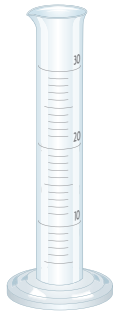
graduated
cylinder

drinking
glass

13. Suppose that tap water, flowing from a faucet at a constant rate, is used to fill these containers. Match each of the following graphs with the appropriate container. Justify your choices.

a)

b)

c)

d)

14. The following table shows the amount of greenhouse gases emitted by fossil fuel production in Canada from 1990 to 2006. During which period was the amount emitted increasing at the greatest rate? Justify your decision.

Year	Greenhouse Gases from Fossil Fuel Production (megatonnes)
1990	103
1995	127
2000	151
2003	161
2006	158

15. At the 2002 Olympics, speed skater Cindy Klassen of Winnipeg, Manitoba, finished out of the medals. Four years later, at the 2006 Olympics, she won five medals in women's speed skating, including gold in the 1500 m race. Compare her speeds in the 1500 m race in 2002 and 2006. In which portion of the race did her speed differ the most?

Games	Age	City	Rank	Time	300 m	700 m	1100 m
2002 Winter	22	Salt Lake	4	1:55.59	25.65	54.17	1:24.09
2006 Winter	26	Torino	1	1:55.27	25.42	53.83	1:23.50

Closing

16. a) When comparing rates, when is an estimate sufficient and when is a precise solution needed? Use specific examples to support your answer.

b) When comparing rates, when is a graphing strategy a good approach and when is a numerical strategy better? Use specific examples to support your answer.

Extending

17. Water is poured into a container like the one to the right at a constant rate. Sketch a graph of depth versus time to represent this situation.

18. Scientists estimate that the processing power of the human brain is equivalent to about 100 million MIPS (million computer instructions per second).

a) In 2005, a typical home computer could process about 7000 MIPS. About how many of these computers were equivalent to the processing power of the brain?

b) Determine the processing power of a computer you use. How many of these computers are equivalent to the processing power of the brain?
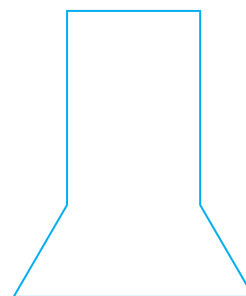
History | Connection

Ivy Granstrom

Ivy Granstrom was born in 1911 in Glace Bay, Nova Scotia, but lived most of her life in Vancouver, British Columbia. She was blind from birth. For 76 consecutive years, she participated in the annual English Bay Polar Bear swim and was affectionately known as the Queen of the Polar Bears. When she was 64 years old, she was struck by a car. Doctors told her that she would always need a wheelchair. However, she began her own rehab program, walking, jogging, and then running. Soon after, she participated in blind sports competitions for the visually impaired. She established herself as one of world's fastest runners in the Masters Division, setting 25 world records for athletes aged 60 years and older, competing against sighted athletes. She raced until 2001 with Paul Hoeberigs, who ran behind, tethered by a cord and calling out directions. Ivy was a Sports B.C. Athlete of the Year in 1982, inducted into the Terry Fox Hall of Fame in 2001, and made a Member of the Order of Canada in 1989. In 2009, she still held the world record in the 1500 m race for women aged 85 and over. Her time of 10:33.40 was established in 1997. She passed away in 2004.

A. If Granstrom ran the mile race at her world record rate for the 1500 m race, could she break the record, which was 11:03.11 in 2009? Justify your answer.

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