7.1 Exploring Quadratic Relations

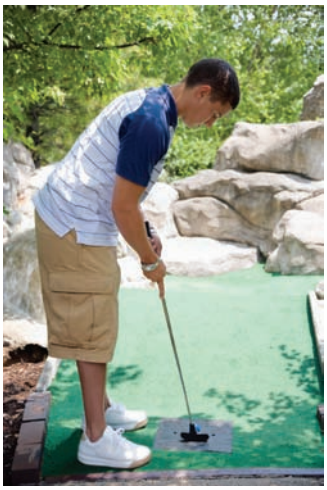
YOU WILL NEED

- graphing technology

GOAL

Determine the characteristics of quadratic relations.

EXPLORE the Math



A moving object that is influenced by the force of gravity can often be modelled by a **quadratic relation** (assuming that there is no friction). For example, on one hole of a mini-golf course, the ball rolls up an incline after it is hit, slowing all the way due to gravity. If the ball misses the hole, it rolls back down the incline, accelerating all the way. If the initial speed of the ball is 6 m/s, the distance of the ball from its starting point in metres, y, can be modelled by the quadratic relation

$$y = -2.5x^2 + 6x$$

where x is the time in seconds after the ball leaves the putter.

quadratic relation
A relation that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$; for example, $y = 4x^2 + 2x + 1$

? How does changing the coefficients and constant in a relation that is written in the form $y = ax^2 + bx + c$ affect the graph of the relation?

Reflecting

parabola
The shape of the graph of any quadratic relation.

A. Describe the common characteristics of each of the **parabolas** you graphed.

B. Describe any symmetry in your graphs.

C. Are the quadratic relations that you graphed functions? Justify your decision.

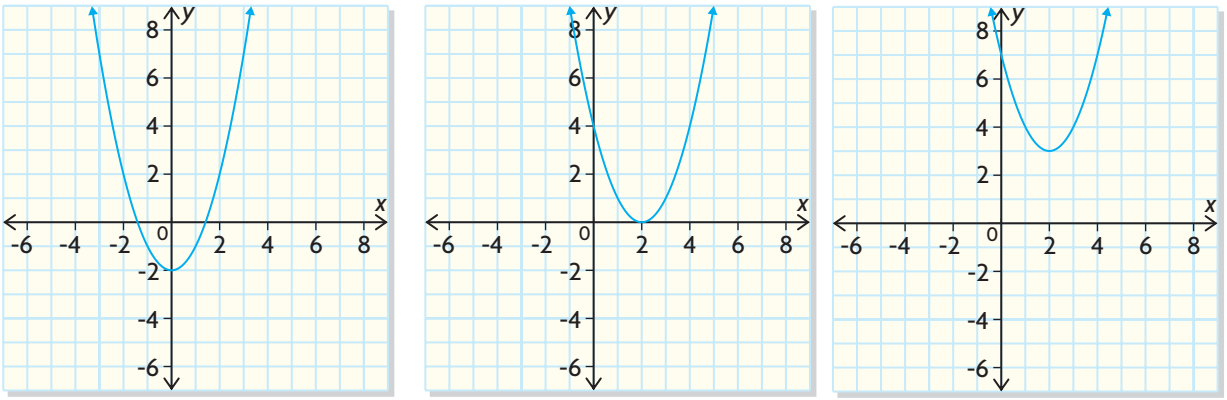
D. What effects do the following changes have on a graph of a quadratic relation?

i) The value of a is changed, but b and c are left constant.

ii) The value of b is changed, but a and c are left constant.

iii) The value of c is changed, but a and b are left constant.

E. The graphs of three quadratic relations are shown. Predict possible values of a, b, and c in the equation for each graph.



In Summary

Key Ideas

- The degree of all quadratic functions is 2.
- The standard form of a quadratic function is

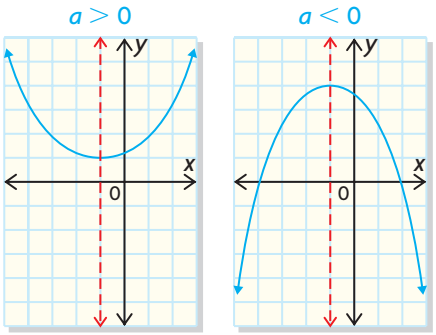
$$y = ax^2 + bx + c$$

where $a \neq 0$.

- The graph of any quadratic function is a parabola with a single vertical line of symmetry.

Need to Know

- A quadratic function that is written in standard form, $y = ax^2 + bx + c$, has the following characteristics:
  - The highest or lowest point on the graph of the quadratic function lies on its vertical line of symmetry.
  - If a is positive, the parabola opens up. If a is negative, the parabola opens down.

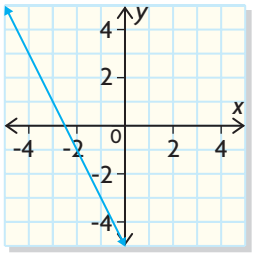


  - Changing the value of b changes the location of the parabola's line of symmetry.
  - The constant term, c, is the value of the parabola's y-intercept.

FURTHER Your Understanding

1. Which graphs appear to represent quadratic relations? Explain.

a)



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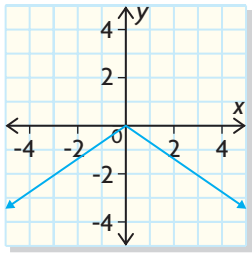

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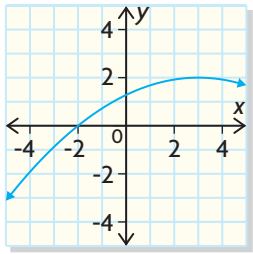

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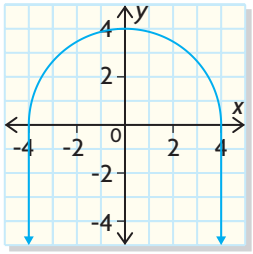

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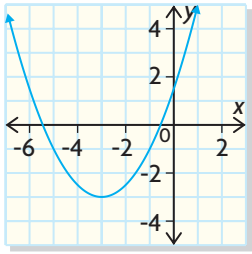

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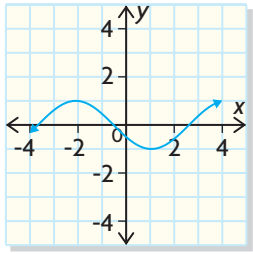

2. Which of the following relations are quadratic? Explain.

a) $y = 2x - 7$ **d)** $y = x^2 - 5x - 6$

b) $y = 2x(x + 3)$ **e)** $y = 4x^3 + x^2 - x$

c) $y = (x + 4)^2 + 1$ **f)** $y = x(x + 1)^2 - 7$

3. State the y-intercept for each quadratic relation in question 2.

4. Explain why the condition $a \neq 0$ must be stated when defining the standard form, $y = ax^2 + bx + c$.

5. Each of the following quadratic functions can be represented by a parabola. Does the parabola open up or down? Explain how you know.

a) $y = x^2 - 4$ **c)** $y = 9 - x + 3x^2$

b) $y = -2x^2 + 6x$ **d)** $y = -\frac{2}{3}x^2 - 6x + 1$

6. Each table of values lists points in a quadratic relation. Decide, without graphing, the direction in which the parabola opens.

a)

x	-4	-3	-2	-1	0	1
y	12	5	0	-3	-4	-3

b)

x	0	1	2	3	4	5
y	-13	-3	3	5	3	-3

c)

x	-5	-4	-3	-2	-1	0
y	3.0	-0.5	-3.0	-4.5	-5.0	-4.5

d)

x	0	1	2	3	4	5
y	-4	19	40	59	76	91