# 6.5 Optimization Problems II: Exploring Solutions

**YOU WILL NEED**

- graphing technology OR graph paper, ruler, and coloured pencils

**GOAL**

Explore the feasible region of a system of linear inequalities.

## *EXPLORE the Math*

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

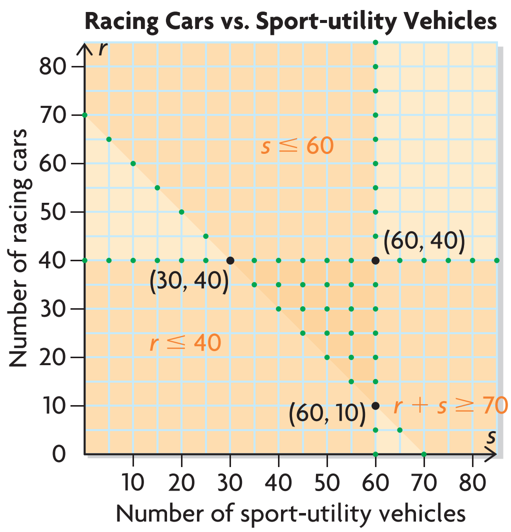
There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

The following model represents this situation. The feasible region of the graph represents all the possible combinations of racing cars ($r$) and sport-utility vehicles ($s$).

Variables:
Let $s$ represent the number of sport-utility vehicles.
Let $r$ represent the number of racing cars.
Let $C$ represent the cost of production.

Restrictions:
$s \in W$, $r \in W$

Constraints:
$s \geq 0$
$r \geq 0$
$r \leq 40$
$s \leq 60$
$r + s \geq 70$




Objective function to optimize:
$C = 12s + 8r$

**?** How can you use patterns in the feasible region to predict the combinations of sport-utility vehicles and racing cars that will result in the minimum and maximum values of the objective function?

## Reflecting

**A.** With a partner, discuss the pattern in the value of $C$ throughout the feasible region. Is the pattern what you expected? Explain.

**B.** As you move from left to right across the feasible region, what happens to the value of $C$?

**C.** As you move from the bottom to the top of the feasible region, what happens to the value of $C$?

**D.** What points in the feasible region result in each **optimal solution**?

**i)** the maximum possible value of $C$

**ii)** the minimum possible value of $C$

**E.** Explain how you could verify that your solutions from part D satisfy each constraint in the model.

**optimal solution**

A point in the solution set that represents the maximum or minimum value of the objective function.

### In Summary

#### Key Ideas

- The value of the objective function for a system of linear inequalities varies throughout the feasible region, but in a predictable way.
- The optimal solutions to the objective function are represented by points at the intersections of the boundaries of the feasible region. If one or more of the intersecting boundaries is not part of the solution set, the optimal solution will be nearby.

#### Need to Know

- You can verify each optimal solution to make sure it satisfies each constraint by substituting the values of its coordinates into each linear inequality in the system.
- The intersection points of the boundaries are called the vertices, or corners, of the feasible region.

## *FURTHER Your Understanding*

**1.** Where might you find the maximum and minimum solutions to each objective function below? Explain how you know.

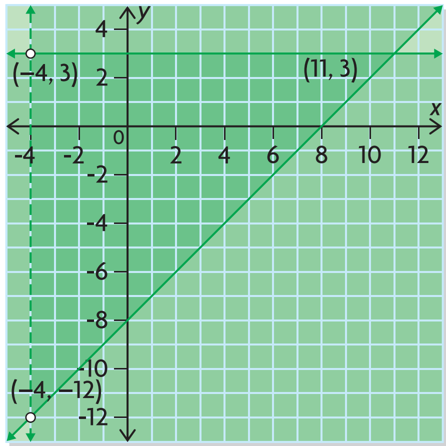
**a) Model A**

Restrictions:
$x \in R, y \in R$

Constraints:
$x > -4$
$x - y \leq 8$
$y \leq 3$

Objective function:
$T = 2x + 5y$

Graph of Model A:



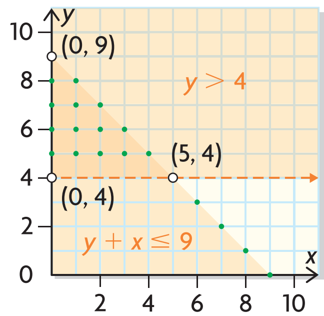

**b) Model B**

Restrictions:
$x \in W, y \in W$

Constraints:
$x \geq 0$
$y \geq 0$
$y > 4$
$y + x \leq 9$

Objective function:
$N = 3x - 2y$

Graph of Model B:

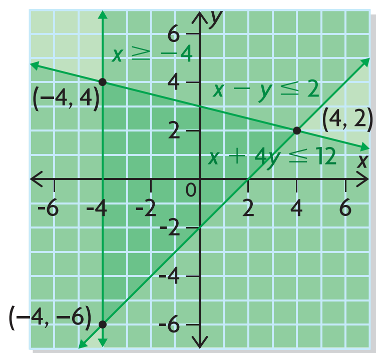



**2.** Consider the model below. What point in the feasible region would result in the minimum value of the objective function? How could you have predicted this from examining the objective function?

Restrictions:
$x \in R, y \in R$

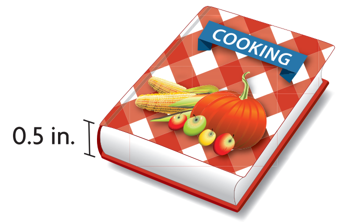
Constraints:
$x + 4y \leq 12$
$x - y \leq 2$
$x \geq -4$

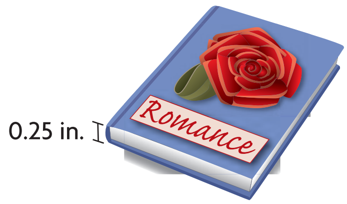
Objective function:
$P = x - y$



3. Meg is building a bookshelf to display her cookbooks and novels.
- She has no more than 50 cookbooks and no more than 200 novels.
- She wants to display at least 2 novels for every cookbook.
- The cookbook spines are about half an inch wide, and the novel spines are about a quarter of an inch wide.

Meg wants to know how long to make the bookshelf.




The following model represents this situation.

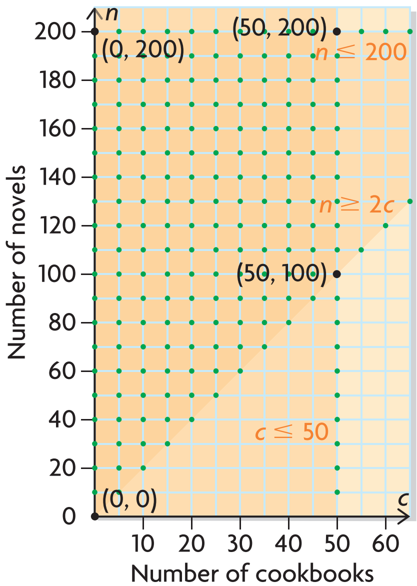
Let $c$ represent the number of cookbooks.
Let $n$ represent the number of novels.
Let $W$ represent the width of the bookshelf.

Restrictions:
$c \in \mathrm{W}, n \in \mathrm{W}$

Constraints:
$c \geq 0$
$n \geq 0$
$c \leq 50$
$n \leq 200$
$n \geq 2c$

Objective function:
$W = 0.5c + 0.25n$

**Collection of Novels and Cookbooks**




**a)** Which point in the feasible region represents the greatest number of books (both cookbooks and novels) that Meg could have? Explain how you know.
**b)** Can she display the same number of cookbooks as novels? Explain.
**c)** What point represents the most cookbooks and the fewest novels?
**d)** What point represents the number of cookbooks that would require the longest shelf? How long would the shelf have to be?
**e)** What point represents the number of cookbooks that would require the shortest shelf?