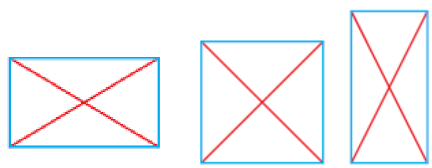
Chapter 1 Questions

1) Make a conjecture about the product of two even numbers. Show three calculations to support your conjecture. multiply

$4 \times 6 = 24$
$-2 \times -16 = 32$
$14 \times -8 = -112$

Conjecture: The product of two even numbers is even

2) Joseph studied the diagonals of the rectangles below to look for a pattern. Make a conjecture about the diagonals of rectangles. What did you do in order to make this conjecture?



Possible answers:
① Diagonals of a rectangle bisect each other. cut each other exactly in half
To make this conjecture, I used a ruler and found it out.
② Diagonals of a rectangle divide the rectangle into 4 sections.
To make this conjecture, I looked at the pattern and saw it was always true!

3) Lucy travelled to London for a week and it rained every day she was there. As a result, she made the following conjecture: "It rains every day in London." Based on the available evidence, is Lucy's conjecture reasonable? Yes, based on her evidence it is reasonable!
What evidence would it take to prove Lucy's conjecture false?
One day without rain (this would be a counterexample)

4) Jerrod examined these multiples of 9: 27, 45, 18, 63, 81, 108, 216, 423. He claimed that the sum of the digits in any multiple of 9 will add to 9 (for example, for 27, 2+7=9). Do you agree or disagree? Justify your decision.

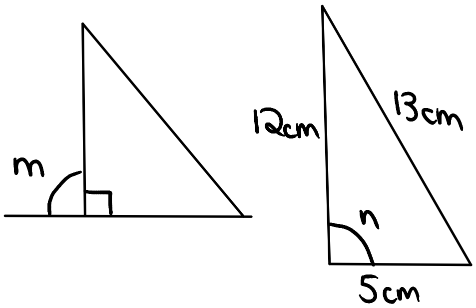
Seems true, however a counterexample could be 99.
It is a multiple of 9 $(9 \times 11 = 99)$
but its digits don't add to 9 $(9+9=18)$
Lots of counterexamples: 189, 198, 279, 288, 297, ...

5) Weight-lifting builds muscle. Muscle makes you strong. Strength improves

balance. Tanner lifts weights. What can be deduced about Tanner?

He build muscle.
or He is stronger.
or His balance is improved.

6) Prove that m and n are equal. Show how you know.



$m = 90^\circ$ (angles on a line $= 180^\circ$)

$n = 90^\circ$ if $a^2 + b^2 = c^2$ ← so... $m = n$

does $5^2 + 12^2 = 13^2$?

$25 + 144 = 169$

$169 = 169$ Yes

7) Conjecture: The product of two odd numbers is always odd.

a) Show inductively, using three examples, that the product is always odd

$3 \times 7 = 21$
$-5 \times -11 = 55$
$-9 \times 13 = -117$

b) **Prove** the conjecture to be true.

Let $2n+1$ = first odd #
Let $2m+1$ = second odd #

$(2n+1)(2m+1)$

$= 4nm + 2n + 2m + 1$
$= 2(2nm + n + m) + 1$
$2($ anything $) + 1$ is always odd, so the product is odd!

8) Prove that whenever you square an even integer, the result is always even.

Let $2n$ = even #

$(2n)^2$

$= (2n)(2n)$
$= 4n^2$
$= 2(2n^2)$
2(anything) is always even, so the result is even!

9) Andy created this step-by-step number trick.
   Choose a number.
   Multiply by 6.
   Add 9.
   Divide by 3.
   Add 5.
   Divide by 2.
   Subtract 4.

a) Show inductively, using two examples, that any number you choose will also be the final result.

$3 \cdot 6 = 18 + 9 = 27 \div 3 = 9 + 5 = 14 \div 2 = 7 - 4 = 3$

$-5 \cdot 6 = -30 + 9 = -21 \div 3 = -7 + 5 = -2 \div 2 = -1 - 4 = -5$

b) **Prove** deductively that any number you choose will also be the final result.

Step	Result
Choose #	n
Multiply by 6	$6n$
Add 9	$6n+9$
Divide by 3	$2n+3$
Add 5	$2n+8$
Divide by 2	$n+4$
Subtract 4	n

____ 10 Which type of reasoning does the following statement demonstrate?

Over the past 11 years, a tree has produced peaches each year.
Therefore, the tree will produce peaches this year.

a) inductive reasoning ← based on pattern
b) deductive reasoning
c) neither inductive nor deductive reasoning

____ 11 Which type of reasoning does the following statement demonstrate?

All birds have feathers.
Robins are birds.
Therefore, robins have feathers.

a) inductive reasoning
b) neither inductive nor deductive reasoning
c) deductive reasoning ← like a proof.

____ 12 What type of error, if any, occurs in the following deduction?

All swimmers can swim one kilometre without stopping. ← not true!
Joan is a swimmer.
Therefore, Joan can swim one kilometre without stopping.

a) a false assumption or generalization
b) an error in reasoning
c) an error in calculation
d) There is no error in the deduction.

____ 13 Determine the unknown term in this pattern.

×2 ×2 ×2
1, 2, 4, 8, 16, 32, 64

a) 6
b) 12
c) 8
d) 10

____ 14 Determine the unknown term in this pattern.

1, 1, 2, 3, 5, ____, 13, 21

a) 6
b) 7
c) 8
d) 9

1+1=2
1+2=3
2+3=5
3+5=8

Choose the next figure in this sequence.

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5 Figure 6

a)

b)

c)

d)

Short Answer

What number should appear in the centre of Figure 4? (1 mark)

4+4+4=12 4+2+5=11 7+2+7=16 6+4+6=16

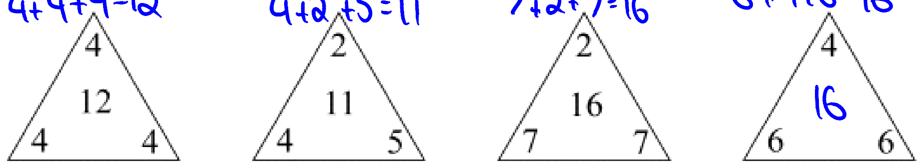


Figure 1 Figure 2 Figure 3 Figure 4

Answer: 16

17 Complete the conclusion for the following deductive argument: (1 mark)

If an even integer is not divisible by 4, then half the number is an odd number.
14 is not divisible by 4, therefore, ...

Answer: therefore half of 14 is an odd number

18 Examine the following example of deductive reasoning. Why is it faulty? (1 mark)

Given: All islands are surrounded by water. Whales are surrounded by water.
Deduction: Whales are islands.

Answer: Islands are not exclusively the only thing surrounded by water

What error occurs in the following deduction? Briefly justify your answer. (2 marks)

Let $x = y$.

$$\begin{aligned} x^2 &= xy \\ x^2 + x^2 &= x^2 + xy \\ 2x^2 &= x^2 + xy \\ 2x^2 - 2xy &= x^2 + xy - 2xy \\ 2x^2 - 2xy &= x^2 - xy \\ 2(x^2 - xy) &= 1(x^2 - xy) \\ 2 &= 1 \end{aligned}$$

they divided by $x^2 - xy$, which is equal to 0

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they divided by $x^2 - xy$, which is equal to 0 if $x = y$

Answer: ____

____

Alison created a number trick in which she always ended with the original number. When Alison tried to prove her trick, however, it did not work. In which step does the calculation error occur? What is the error? (2 marks)

should be	n	Use n to represent any number.
	$n + 4$	Add 4.
$2n+8$	$2n + 4$	Multiply by 2.
	$2n + 8$	Add 4.
	$n + 4$	Divide by 2.
	$n - 1$	Subtract 5.

Answer: n+4 times two is 2n+8

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